Unit 4 Packet – Exponential and Logarithmic Functions
Pre-Calculus Honors

Name: Key

What is the difference between a power function and exponential function?

POWER

$y = kx^a$

The base is a variable & the exponent is a constant.

EXPONENTIAL

$y = a \cdot b^x$

initial amount (or value)

Base $b > 0, b \neq 1$.

variable in the exponent

Exponential functions are continuous for all real numbers.

Domain is always $\mathbb{R}$, $(-\infty, \infty)$.

Functions	Exponential (yes or no)	If no, Why?	If yes, Initial Value	If yes, Base
$f(x) = 3^x = 1 \cdot 3^x$	yes		1	3
$f(x) = 6x^4 = \frac{6}{x^{-4}}$	no	exponent is NOT a variable		
$f(x) = -2(1.5)^x$	yes		-2	1.5
$f(x) = 7(2)^{-x}$	yes		7	1/2
$f(x) = 5(6)^{\pi}$	no	exponent is NOT a variable		

$f(x) = 7\left[(2)^{-1}\right]^x = 7\left(\frac{1}{2}\right)^x$

? Mult. ↓ ↓ or ÷ ↑

$y = (\text{initial})(\text{base rate})^x$

"a" (0, a) "b"

Making Exponential Functions from a table of values:

x	g(x)
-2	128
-1	32
0	8
1	2
2	1/2

$y = 8 \cdot (1/4)^x$ or $y = 8 \cdot 4^{-x}$

← a = 8

b = 1/4

$\frac{1/2}{2} = 1/4$ $\frac{8}{32} = 1/4$

$\frac{2}{8} = 1/4$ $\frac{32}{128} = 1/4$

x	h(x)
-2	4/9
-1	4/3
0	4
1	12
2	36

$y = 4 \cdot (3)^x$

← a = 4

b = 3

$\frac{36}{12} = 3$ $\frac{4}{4/3} = \frac{4}{1} \cdot \frac{3}{4} = 3$

$\frac{12}{4} = 3$ $\frac{4/3}{4/9} = \frac{4}{3} \cdot \frac{9}{4} = 3$

x	f(x)
-2	ab^{-2}
-1	ab^{-1}
0	a
1	ab
2	ab^2

$y = a \cdot b^x$

← a = a

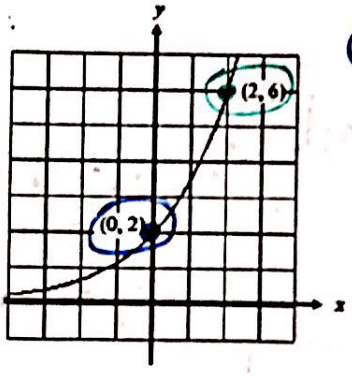
b = b

$\frac{ab}{ab} = b$ $\frac{a}{ab^{-1}} = b$

$\frac{ab}{a} = b$ $\frac{ab^{-1}}{ab^{-2}} = b^{-1+2} = b^1$

Making Exponential Functions from a graph:

Determine the formula for the graph and the two points



$(x,y) \rightarrow$ pt. on curve

$y = a \cdot b^x$

① Use y-int: (0, 2)

$2 = a \cdot b^0$

$2 = a(1)$

$a = 2$

② Use (2, 6):

$6 = 2 \cdot b^2$

$\frac{6}{2} = \frac{2}{2} \cdot b^2$

$\sqrt{3} = \sqrt{b^2}$

$b = \sqrt{3}$ ← b cannot be neg.!

③ $y = 2 \cdot (\sqrt{3})^x$

$y = 2 \cdot (3^{1/2})^x$

$y = 2 \cdot 3^{x/2}$

Transformation Formula: $y = a(b)^{x-h} + k$

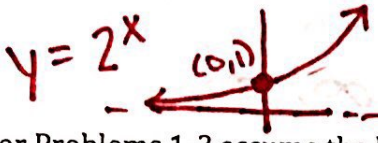
H.A.: $y = k$

+k: up k

-k: down k

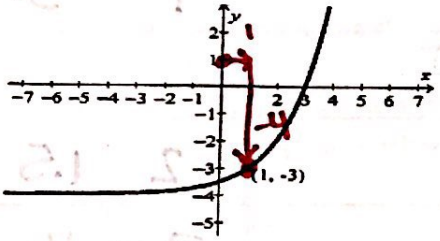
+h: left h

-h: right h



For Problems 1-3 assume the base of the function is 2.

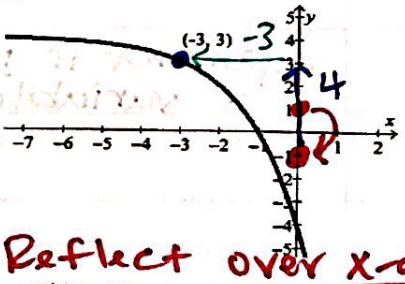
1. $f(x) = 2^{x-1} - 4$



Right 1

Down 4

2. $f(x) = -2^{x+3} + 4$



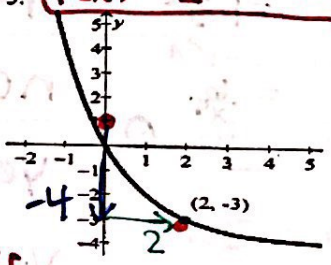
Reflect over x-axis

Up 4

Left 3

3. $f(x) = 2^{-(x-2)} - 4$

$f(x) = 2^{-x+2} - 4$



Reflect over y

Down 4

Right 2

Sketch the graph of the exponential function

a) $y = g(x) = 3^x + 0$

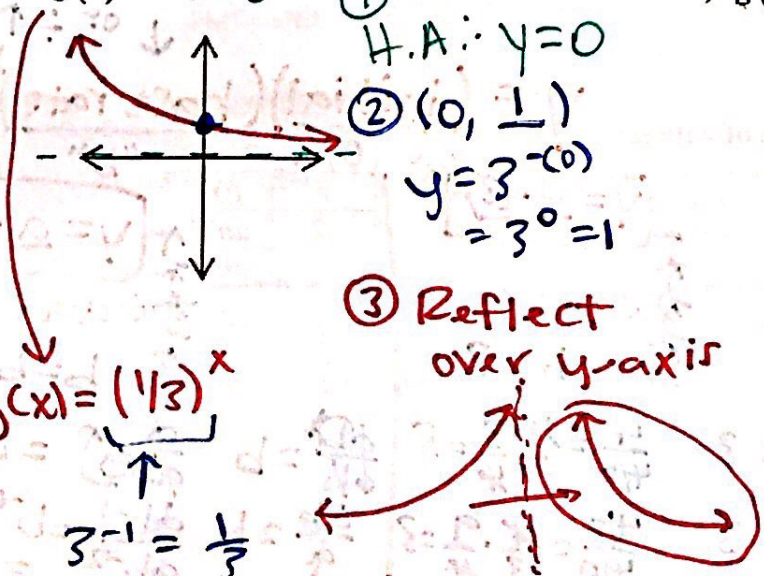
① H.A.: $y = 0$

② $(0, 1)$

$y = 3^{-(0)}$

$= 3^0 = 1$

③ Reflect over y-axis

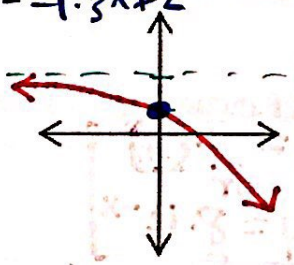


$g(x) = (1/3)^x$

$3^{-1} = \frac{1}{3}$

b) $g(x) = -3^x + 2$

$= -1 \cdot 3^x + 2$



① H.A.: $y = 2$

② $(0, 1)$

$g(0) = -3^0 + 2$

$= -1 \cdot 1 + 2$

$= -1 + 2$

$= 1$

③ Reflect over x-axis

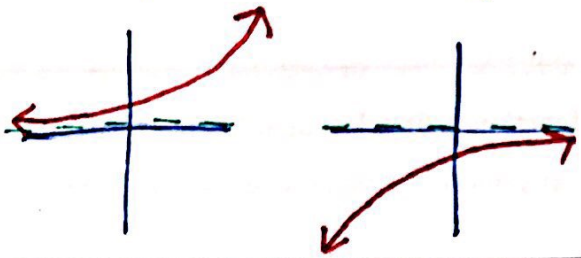
Exponential Growth and Decay:

$y = a(b)^x$

Basic Shapes:

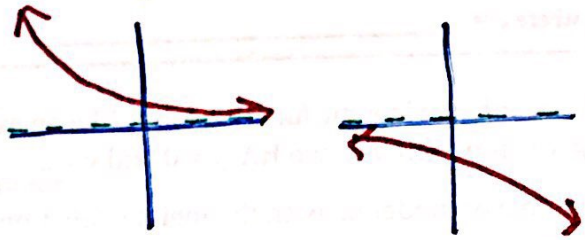
Growth: $b > 1$

always increasing



Decay: $0 < b < 1$

always decreasing



	Function $y = \square \cdot \square^{x} \pm k$	Initial Amount a	Y-Intercept $(0, \_)$	Growth or Decay Factor b	Growth or Decay?	End Behavior
①	$f(x) = 2^{-2x}$					
②	$g(x) = 6(\frac{1}{4})^{-3x} + 5$					
③	$h(x) = 2(\frac{7}{5})^{2x} - 4$ $= 2(\frac{49}{25})^{x} - 4$	2	$h(0) = 2(\frac{7}{5})^{0} - 4$ $= 2 \cdot 1 - 4$ $(0, -2)$	$\frac{49}{25}$	$\frac{49}{25} > 1$ Growth	L: -4 R: ∞
④	$h(x) = 4(.25)^{-x} - 4$					

Sketch each graph:

①

②

③

① H.A.: $y = -4$

② $(0, -2)$

③ growth $\Rightarrow$ inc.

④