DIAGNOSTIC TEST ANSWER SHEET

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____

18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____

35. _____
36. _____
37. _____
38. _____
39. _____
40. _____
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____
47. _____
48. _____
49. _____
50. _____

3.1 Getting Started!

Taking the Diagnostic Test helps you assess your strengths and weaknesses as you begin preparing for the AP Calculus AB exam. The questions in the Diagnostic Test contain both multiple-choice and open-ended questions. They are arranged by topic and designed to review concepts tested on the AP Calculus AB exam. All questions in the diagnostic test should be done without the use of a graphing calculator, except in a few cases where you need to find the numerical value of a logarithmic or exponential function.

3.2 Diagnostic Test

Chapter 5

1. Write an equation of a line passing through the origin and perpendicular to the line $5x - 2y = 10$.

2. Solve the inequality $3|2x - 4| > 6$ graphically. Write the solution in interval notation.

3. Given $f(x) = x^2 + 1$, evaluate $\dfrac{f(3+h) - f(3)}{h}$.

4. Solve for x: $4 \ln x - 2 = 6$.

5. Given $f(x) = x^3 - 8$ and $g(x) = 2x - 2$, find $f(g(2))$.

Chapter 6

6. A function f is continuous on $[-2, 0]$ and some of the values of f are shown below.

x	-2	-1	0
f	4	b	4

If $f(x) = 2$ has no solution on $[-2, 0]$, then b could be.

(A) 3

(B) 2

(C) 1

(D) 0

(E) –2

7. Evaluate $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 - 4}}{2x}$.

8. If
$$h(x) = \begin{cases} \sqrt{x} & \text{if } x > 4 \\ x^2 - 12 & \text{if } x \le 4 \end{cases} \quad \text{find } \lim\limits_{x \to 4} h(x).$$

9. If $f(x) = |2xe^x|$, what is the value of $\lim\limits_{x \to 0^+} f'(x)$?

Chapter 7

10. If $f(x) = -2 \csc(5x)$, find $f'\left(\dfrac{\pi}{6}\right)$.

11. Given the equation $y = (x+1)(x-3)^2$, what is the instantaneous rate of change of y at $x = -1$?

12. What is $\lim\limits_{\Delta x \to 0} \dfrac{\tan\left(\dfrac{\pi}{4} + \Delta x\right) - \tan\left(\dfrac{\pi}{4}\right)}{\Delta x}$?

Chapter 8

13. The graph of f is shown in Figure D-1. Draw a possible graph of f' on (a, b).
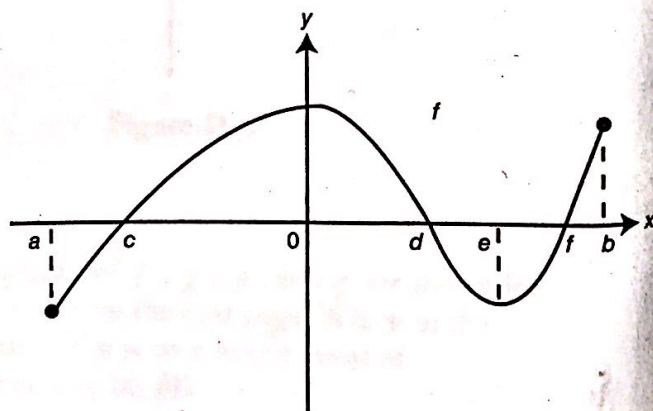

Figure D-1

14. The graph of the function g is shown in Figure D-2. Which of the following is true for g on (a, b)?

   I. g is monotonic on (a, b).

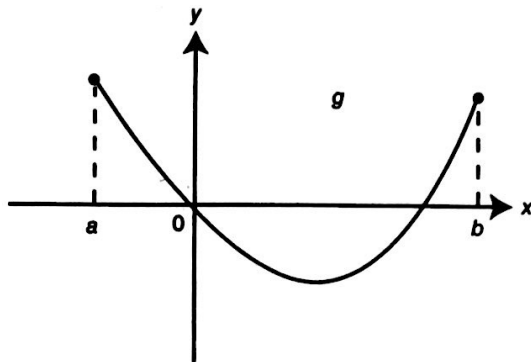   II. g' is continuous on (a, b).

   III. $g'' > 0$ on (a, b).

Figure D-2

15. The graph of f is shown in Figure D-3 and f is twice differentiable, which of the following statements is true?
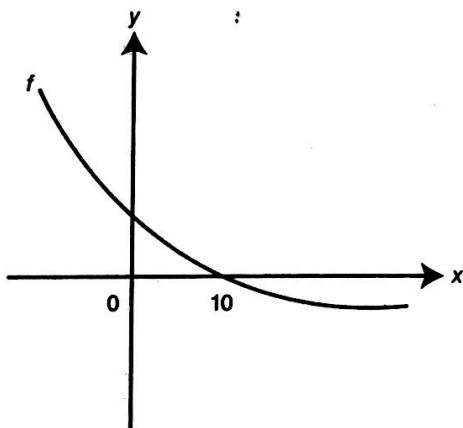

Figure D-3

(A) $f(10) < f'(10) < f''(10)$

(B) $f''(10) < f'(10) < f(10)$

(C) $f'(10) < f(10) < f''(10)$

(D) $f'(10) < f''(10) < f(10)$

(E) $f''(10) < f(10) < f'(10)$

16. The graph of f', the derivative of f, is shown in Figure D-4. At what value(s) of x is the graph of f concave up?
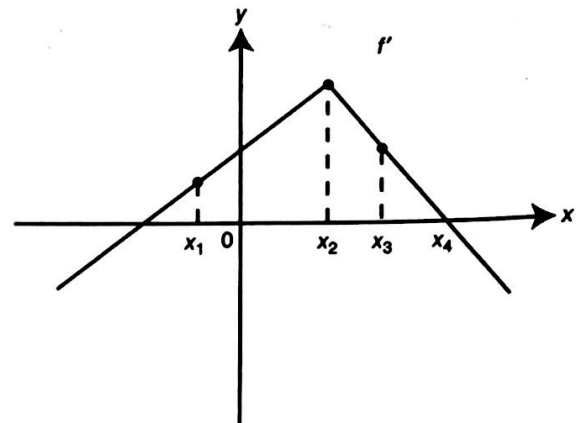

Figure D-4

17. How many points of inflection does the graph of $y = \sin(x^2)$ have on the interval $[-\pi, \ \pi]$?

18. If $g(x) = \displaystyle\int_{a}^{x} f(t)\,dt$ and the graph of f is shown in Figure D-5, which of the graphs in Figure D-6 on the next page is a possible graph of g?
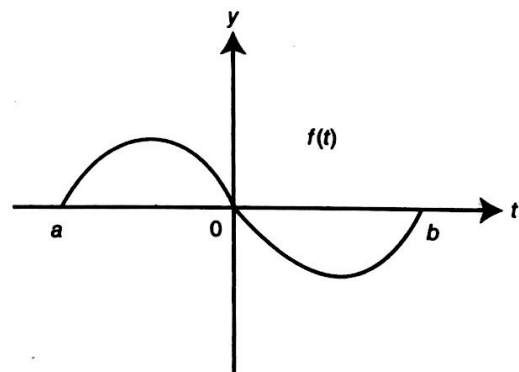

Figure D-5

19. The graphs of f', g', p', and q' are shown in Figure D-7 on the next page. Which of the functions f, g, p, or q have a point of inflection on (a, b)?
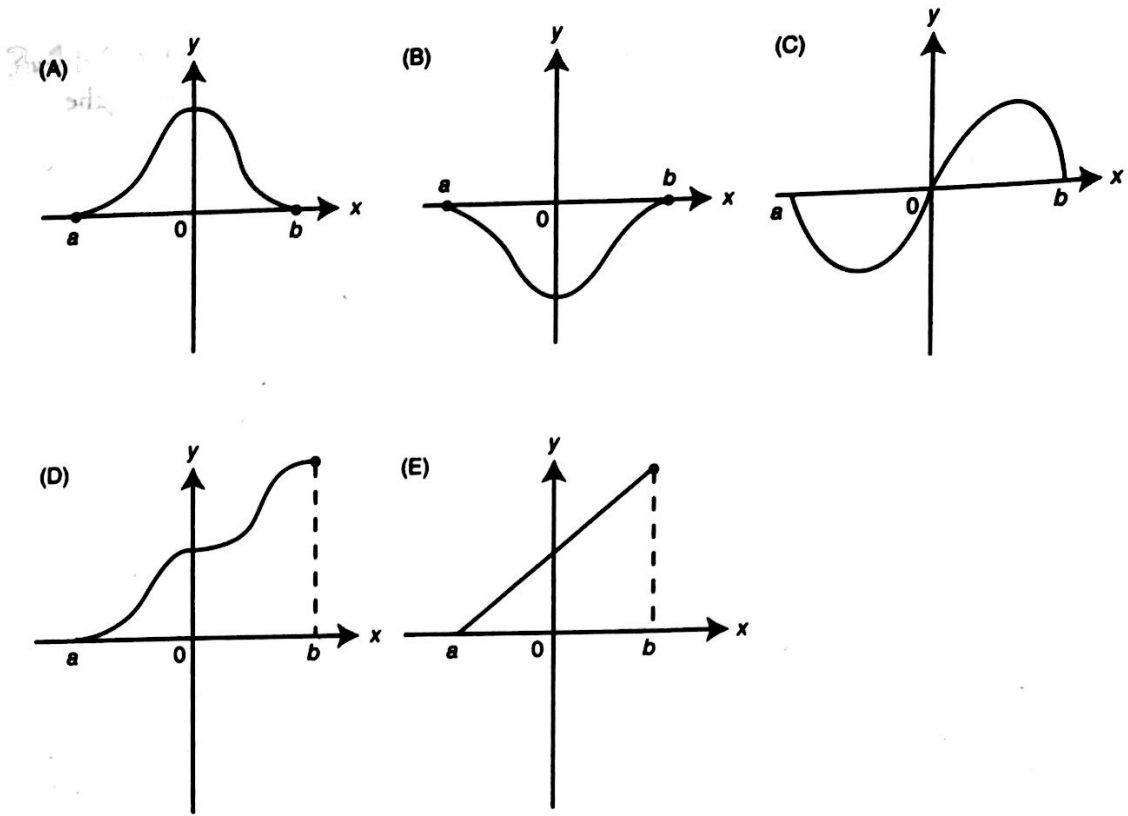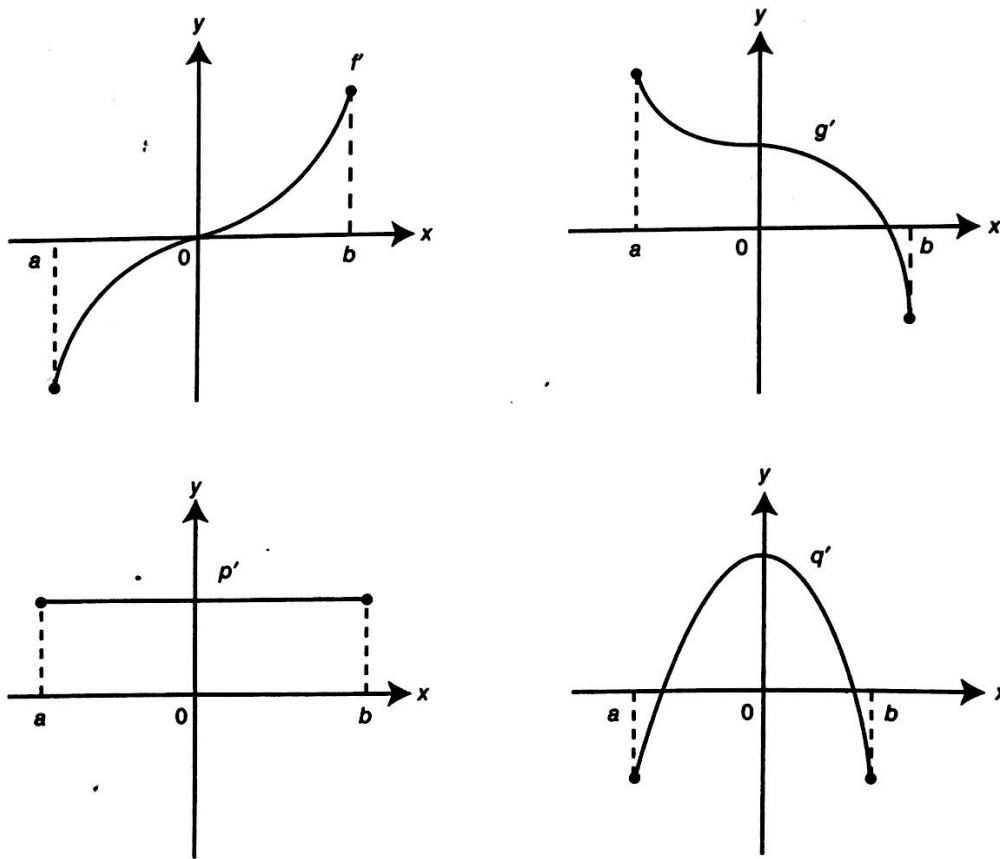
(A)

(B)

(C)

(D)

(E)

Figure D-6

Figure D-7

Chapter 9

20. When the area of a square is increasing four times as fast as the diagonals, what is the length of a side of the square?

21. If $g(x) = |x^2 - 4x - 12|$, which of the following statements about g is/are true?

    I. g has a relative maximum at $x = 2$.

    II. g is differentiable at $x = 6$.

    III. g has a point of inflection at $x = -2$.

Chapter 10

22. Given the equation $y = \sqrt{x-1}$, what is an equation of the normal line to the graph at $x = 5$?

23. What is the slope of the tangent to the curve $y = \cos(xy)$ at $x = 0$?

24. The velocity function of a moving particle on the x-axis is given as $v(t) = t^2 - t$. For what values of t is the particle's speed decreasing?

25. The velocity function of a moving particle is $v(t) = \frac{t^3}{3} - 2t^2 + 5$ for $0 \le t \le 6$. What is the maximum acceleration of the particle on the interval $0 \le t \le 6$?

26. Write an equation of the normal line to the graph of $f(x) = x^3$ for $x \ge 0$ at the point where $f''(x) = 12$.

27. At what value(s) of x do the graphs of $f(x) = \frac{\ln x}{x}$ and $y = -x^2$ have perpendicular tangent lines?

28. Given a differentiable function f with $f\left(\frac{\pi}{2}\right) = 3$ and $f'\left(\frac{\pi}{2}\right) = -1$. Using a tangent line to the graph at $x = \frac{\pi}{2}$, find an approximate value of $f\left(\frac{\pi}{2} + \frac{\pi}{180}\right)$.

Chapter 11

29. Evaluate $\int \frac{1 - x^2}{x^2} \, dx$.

30. If $f(x)$ is an antiderivative of $\frac{e^x}{e^x + 1}$ and $f(0) = \ln(2)$, find $f(\ln 2)$.

31. Find the volume of the solid generated by revolving about the x-axis on the region bounded by the graph of $y = \sin 2x$ for $0 \le x \le \pi$ and the line $y = \frac{1}{2}$.

Chapter 12

32. Evaluate $\int_1^4 \frac{1}{\sqrt{x}} \, dx$.

33. If $\int_{-1}^k (2x - 3) \, dx = 6$, find k.

34. If $h(x) = \int_{\pi/2}^x \sqrt{\sin t} \, dt$, find $h'(\pi)$.

35. If $f'(x) = g(x)$ and g is a continuous function for all real values of x, then $\int_0^2 g(3x) \, dx$ is

    (A) $\frac{1}{3} f(6) - \frac{1}{3} f(0)$.

    (B) $f(2) - f(0)$.

    (C) $f(6) - f(0)$.

    (D) $\frac{1}{3} f(0) - \frac{1}{3} f(6)$.

    (E) $3 f(6) - 3 f(0)$.

36. Evaluate $\int_\pi^x \sin(2t) \, dt$.

37. If a function f is continuous for all values of x, which of the following statements is/are always true?

    I. $\int_a^c f(x)dx = \int_a^b f(x)dx$
    $+ \int_b^c f(x)dx$

II. $\displaystyle\int_a^b f(x)dx = \int_a^c f(x)dx$

$\displaystyle\qquad\qquad - \int_c^b f(x)dx$

III. $\displaystyle\int_b^c f(x)dx = \int_b^a f(x)dx$

$\displaystyle\qquad\qquad - \int_c^a f(x)dx$
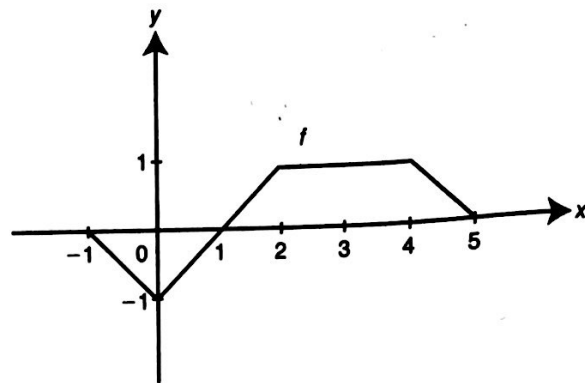
38. If $g(x) = \displaystyle\int_{\pi/2}^x 2\,\sin t\,dt$ on $\left[\dfrac{\pi}{2}, \dfrac{5\pi}{2}\right]$, find the value(s) of x where g has a local minimum.

Chapter 13

39. The graph of the velocity function of a moving particle is shown in Figure D-8. What is the total distance traveled by the particle during $0 \le t \le 6$?
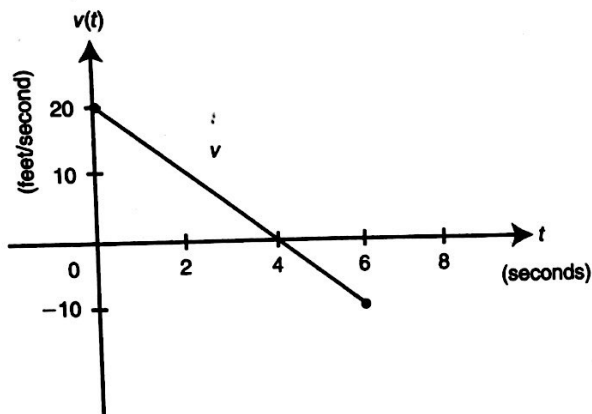

Figure D-8

40. The graph of f consists of four line segments, for $-1 \le x \le 5$ as shown in Figure D-9. What is the value of $\displaystyle\int_{-1}^5 f(x)\,dx$?

Figure D-9

41. Find the area of the region enclosed by the graph of $y = x^2 - x$ and the x-axis.

42. If $\displaystyle\int_{-k}^k f(x)\,dx = 0$ for all real values of k, then which of the graphs in Figure D-10 could be the graph of f? (See next page.)

43. The area under the curve $y = \sqrt{x}$ from $x = 1$ to $x = k$ is 8. Find the value of k.

44. For $0 \le x \le 3\pi$, find the area of the region bounded by the graphs of $y = \sin x$ and $y = \cos x$.

45. Let f be a continuous function on $[0, 6]$ that has selected values as shown below:

x	0	1	2	3	4	5	6
$f(x)$	1	2	5	10	17	26	37

Using three midpoint rectangles of equal widths, find an approximate value of

$$\int_0^6 f(x)\,dx.$$
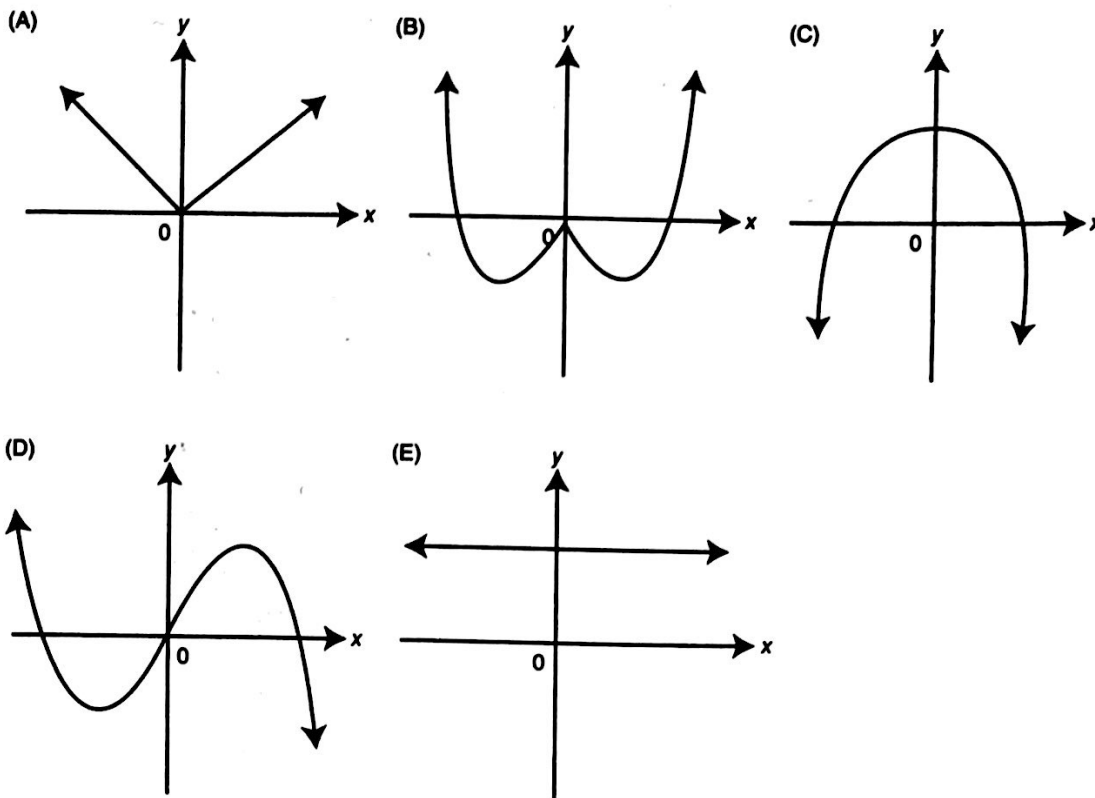
(A)

(B)

(C)

(D)

(E)

Figure D-10

Chapter 14

46. What is the average value of the function $y = e^{-4x}$ on $[-\ln 2, \ln 2]$?

47. If $\dfrac{dy}{dx} = 2\sin x$ and at $x = \pi$, $y = 2$, find a solution to the differential equation.

48. Water is leaking from a tank at the rate of $f(t) = 10\ln(t+1)$ gallons per hour for $0 \le t \le 10$, where t is measured in hours. How many gallons of water have leaked from the tank at exactly after 5 hours?

49. Carbon-14 has a half-life of 5730 years. If y is the amount of Carbon-14 present and y decays according to the equation $\dfrac{dy}{dt} = ky$, where k is a constant and t is measured in years, find the value of k.

50. What is the volume of the solid whose base is the region enclosed by the graphs of $y = x^2$ and $y = x + 2$ and whose cross sections perpendicular to the x-axis are squares?