TEST	SERIES	How it works	Converges	Diverges	Important Information
Geometric Series	$\sum_{n=1}^{\infty} g_1 (r)^{n-1}$	Series where each term is generated by multiplying the previous term by the same value (r is that number; called the ratio & it is constant); <u>the power is variable</u>.	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	**Make sure the series is written properly** before finding the sum: if n = 0 , then r^n; if n = 1, then r^{n-1}. <u>EXAMPLE</u>: $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{2n} = \sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^{n} = \sum_{n=1}^{\infty}\frac{1}{4}\left(\frac{1}{4}\right)^{n-1}$ Sum can be found with: $S = \frac{g_1}{1-r}$
Harmonic Series	$\sum_{n=1}^{\infty}\frac{1}{n}$	If the numerator is something other than 1, factor it outside the sigma.	NEVER	ALWAYS	<u>EXAMPLE</u>: $\sum_{n=1}^{\infty}\frac{5}{n} = 5\sum_{n=1}^{\infty}\frac{1}{n}$
Telescoping Series	$\sum_{n=1}^{\infty}(a_n - a_{n+1})$	Series where it's partial sums collapse (cancel each other out). Expand the series and cancel out to simplify to a partial sum: S_n $\lim_{n\to\infty} S_n$	$\lim_{n\to\infty} S_n < \infty$	$\lim_{n\to\infty} S_n = \infty$	Series is not easy to recognize if not separated into components – use partial fractions for this & you must get subtraction between them. For the type of partial fractions we do, the denominator of your telescoping series <u>must have linear factors</u>. Sum can be found with: $S = \lim_{n\to\infty} S_n$ <u>EXAMPLE</u>: $\sum_{n=1}^{\infty}\frac{1}{2n^2+2n} = \sum_{n=1}^{\infty}\frac{1}{2n(n+1)} = \sum_{n=1}^{\infty}\frac{1}{2n} - \frac{1}{2(n+1)}$ (you can factor our a $\frac{1}{2}$ to make it easier to expand) $S_n = \frac{1}{2} - \frac{1}{2n+2}$, so $\lim_{n\to\infty} S_n = \frac{1}{2}$ therefore $S = \frac{1}{2}$
p-Series	$\sum_{n=1}^{\infty}\frac{1}{n^p}$	<u>The power is constant</u>, but the n changes.	p > 1	$p \leq 1$	If there is an alternating sign in your p-series, you should use the alternating series test.
Nth term Test or Divergence Test	$\sum_{n=1}^{\infty} a_n$	Take $\lim_{n\to\infty} a_n$.	NOT SURE	As long as the limit $\neq 0$.	For series you don't recognize, use this test first. If it's inconclusive, use the p-series or ratio test next.
Alternating Series Test	$\sum_{n=1}^{\infty}(-1)^{n-1} a_n$	<u>Check these 3 things</u>: 1) (w/o negative) Is first term>0? 2) Do terms get successively smaller? $a_1 \geq a_2 \geq a_3 \ldots$ 3) $\lim_{n\to\infty} a_n = 0$?	When all 3 conditions are met	If <u>any</u> conditions fail, we are NOT SURE.	**Use this test only for alternating series.**
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert$; simplify to find r	$\lvert r \rvert < 1$	$\lvert r \rvert > 1$	**If r = 1, we have <u>no conclusion</u>.** Use this test to create a ratio in order to find radius and/or interval of convergence. Use this test whenever you have exponential or factorial expressions.

TEST	SERIES	How it works . . .	Converges	Diverges	Comments
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	<u>Choose a series</u> that you think might match up well with a simplified version of your series. Choose $\frac{1}{n}$ which diverges OR Choose $\frac{1}{n^2}$ which converges. Set up a ratio: $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$ (where b_n is the chosen series) and find the limit L.	If L > 0 & the series you chose converges.	If L > 0 & the series you chose diverges.	This test is easier than the Direct Comparison Test, but it still <u>requires some skill in choosing the series for comparison</u>. Try thinking of what your series might look like if it could be simplified (mainly look at the leading terms) in order to make a choice for b_n. <u>EXAMPLE</u>: $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)=\sum_{n=1}^{\infty}\left(\frac{n-1}{n^2}\right)$ Without the -1 in the numerator (which wouldn't matter as you approach infinity), this series simplifies to $\frac{1}{n}$, so that would be good to use for comparison.
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{a_n}$ You take the nth root to get rid of n's in the exponent.	L < 1	L > 1	**If L = 1, we have <u>no conclusion</u>.** Use this test when you have exponential expressions.
Integral Test	$\sum_{n=1}^{\infty} a_n$	$\int_1^{\infty} f(x)dx$ where $f(x)$ = general term with n being replaced by x.	Series converges if $f(x)$ converges.	Series diverges if $f(x)$ diverges.	This test should only be used when: 1) The series has positive terms. 2) The general expression is one that can be integrated.
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	<u>Choose a series</u> (call it series b) whose terms are bigger than your terms. ($a_1 \le b_1, a_2 \le b_2, a_3 \le b_3, \ldots$)	If b, who's bigger than a converges, so does a. *If b levels off and he's big, then a is certainly going to level off.*	If a, who's smaller than b diverges, so does b. *If a is already out of control, b is certainly going to be out of control.*	Try this test only as a last resort. You have to know what series to choose and how it behaves.