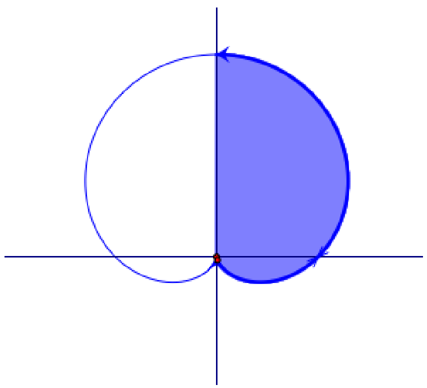
Unit 1 Day 8 Review and Unit Wrap-up

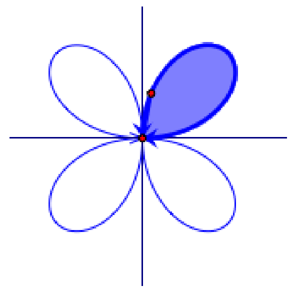
Find the limits (domain) of the specified parts of the graphs (in terms of π)



Right half

$$r = 1 + \sin\theta$$

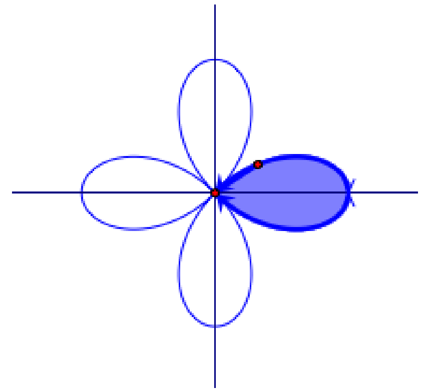
$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



Upper, right leaf

$$r = \sin 2\theta$$

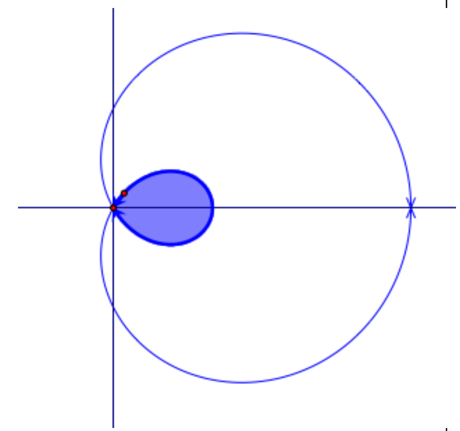
$$\left[0, \frac{\pi}{2}\right]$$



Right leaf

$$r = \cos 2\theta$$

$$\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$



Inner loop

$$r = 1 + 2\cos\theta$$

$$\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$$

HW Questions

Calculator Skills Practice

Using Calculator to analyze graphs

TRACE

FORMAT Parameters
- PolarGC

WINDOW Parameters
- θMin, θMax
- θStep

Finding Polar intersections

DRAW Menu
- Horizontal and Vertical
- Tangent

DRAW

Set $r = 1 + 2\sin\theta$

Set $\theta Min = 0$

$\theta Max = 2\pi$

$\theta Step = \pi / 24$

DRAW Horizontal and Vertical can help visualize and locate tangents.

Option 1:ClrDraw to clear lines

Bring up DRAW menu (2^{nd} PRGM)

Select 3:Horizontal

Use up/down arrows

Press ENTER to fix line

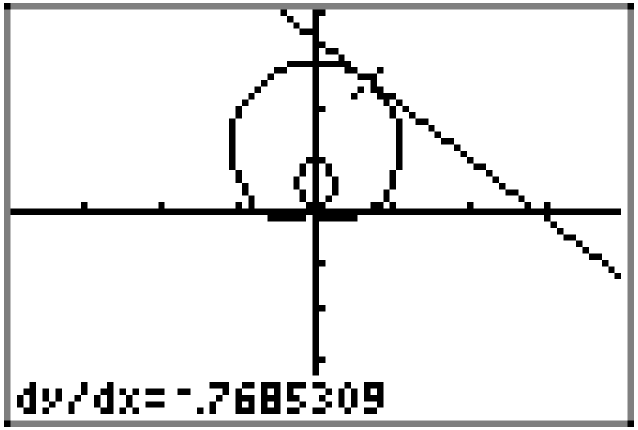
Draw

DRAW Tangent can show tangent at any point and its dy/dx value.

Select 5:Tangent

Use right/left arrows to move around graph to any point

Press ENTER to draw tangent line



Exploring Curves

$$r = \cos 2\theta$$

$$r = \sqrt{\cos 2\theta}$$

Exploring Curves

$$r = 2 + 2\cos\theta$$

$$r = \sqrt{2 + 2\cos\theta}$$

Exploring Curves

$$r = 1 + 2\cos\theta$$

$$r = \sqrt{1 + 2\cos\theta}$$

Exploring Curves

$$r = 2 + 1\cos\theta$$

$$r = \sqrt{2 + 1\cos\theta}$$

Polar Test

- Material
  - Some Old (AB)
  - Some New (Polar)
- Sections
  - Multiple Choice
  - Free-Response
  - Calculator Active and Inactive
- Study Guide on Weebly

Useful identities and integrals

$$\sin(2\theta) = 2\sin\theta\cos\theta$$ ← Memorize this one

Here is what you have been practicing

$$\int \sin^2 A\theta \, d\theta = \int \frac{1-\cos 2A\theta}{2} \, d\theta$$

$$\int \cos^2 A\theta \, d\theta = \int \frac{1+\cos 2A\theta}{2} \, d\theta$$

Good News or Not Good News

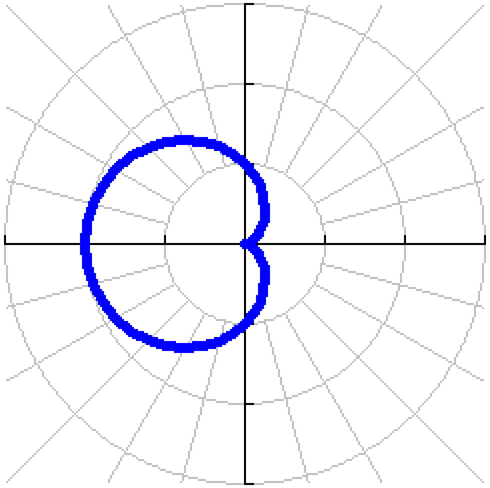
$$\int \sin^2 A\theta \, d\theta = \frac{\theta}{2} - \frac{\sin 2A\theta}{4A} + C$$

$$\int \cos^2 A\theta \, dx = \frac{\theta}{2} + \frac{\sin 2A\theta}{4A} + C$$

If you need these, they will be given on the test.

Using nDeriv

Find the length of the cardioid $r = 1 - \cos\theta$.



$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$$

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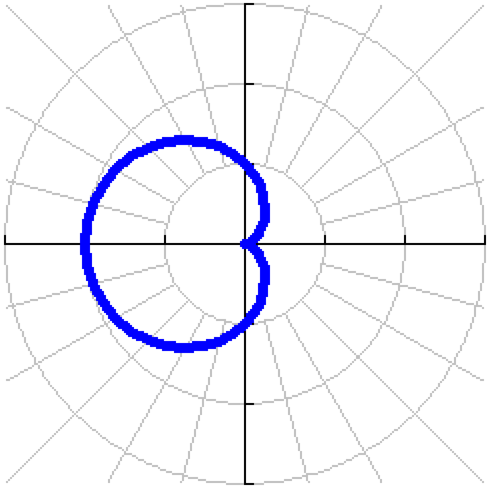
Using nDeriv

Find the length of the cardioid $r=1-\cos\theta$.



$$r1=1-\cos\theta$$

$$r2=nDeriv\left(r1,\theta,\theta\right)$$

$$fnInt\left(\sqrt{r1^2+r2^2},\theta,0,2\pi\right)$$

$$L=8$$

Summary

If free response question

Then write complete integral, including $\frac{dr}{d\theta}$

$$L = \int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}\ d\theta$$

If calculator active

Then you may use fnInt and nDeriv

$$r1 = 1 - \cos\theta$$

$$r2 = nDeriv(r1, \theta, \theta)$$

$$fnInt\left(\sqrt{r1^2 + r2^2}, \theta, 0, 2\pi\right)$$

If calculator inactive

Then simplify before attempting the integral

$$L = \int_0^{2\pi} \sqrt{2 - 2\cos\theta}\ d\theta$$

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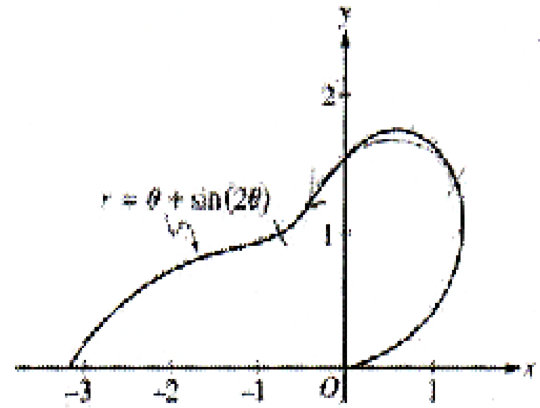
Question 2



The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

$$\text{Area} = \frac{1}{2}\int_0^{\pi} r^2\, d\theta$$
$$= \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2\, d\theta = 4.382$$

$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

$$-2 = r\cos(\theta) = (\theta + \sin(2\theta))\cos(\theta)$$
$$\theta = 2.786$$

$2: \begin{cases} 1: \text{equation} \\ 1: \text{answer} \end{cases}$

Since $\frac{dr}{d\theta} < 0$ for $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, r is decreasing on this interval. This means the curve is getting closer to the origin.

$2: \begin{cases} 1: \text{information about } r \\ 1: \text{information about the curve} \end{cases}$

The only value in $\left[0, \frac{\pi}{2}\right]$ where $\frac{dr}{d\theta} = 0$ is $\theta = \frac{\pi}{3}$.

θ	r
0	0
$\frac{\pi}{3}$	1.913
$\frac{\pi}{2}$	1.571

The greatest distance occurs when $\theta = \frac{\pi}{3}$.

$2: \begin{cases} 1: \theta = \frac{\pi}{3} \text{ or } 1.047 \\ 1: \text{answer with justification} \end{cases}$