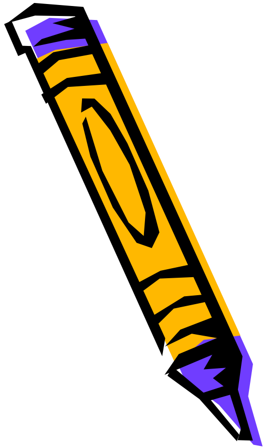
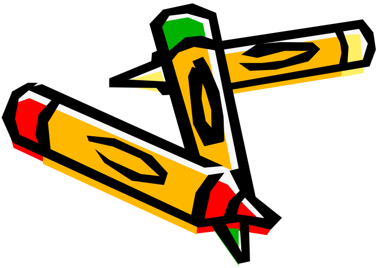
Calculus BC

Unit 1 Day 7



Last Night's HW

Find a partner and pick one homework problem to "teach" together on the board. Everyone gets a turn!!



Calculus BC

NEW Topic
Derivatives of Polar Equations

Let's take a stroll down AB memory land…

Find the derivative:

1) $y = 4x^2 - 3x + 1$

2) $y = \cos x$

3) $y = \left(x^3 - 2x\right)^{12}$

4) $y = \dfrac{x}{x+1}$

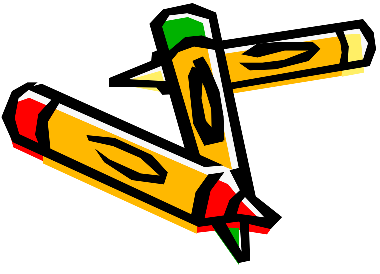
Answers:

1) $y' = 8x - 3$

2) $y' = -\sin x$

3) $y' = 12\left(x^3 - 2x\right)^{11}\left(3x^2 - 2\right)$

4) $y' = \dfrac{1}{\left(x+1\right)^2}$



A derivative is . . .

The slope of a tangent line to the curve.

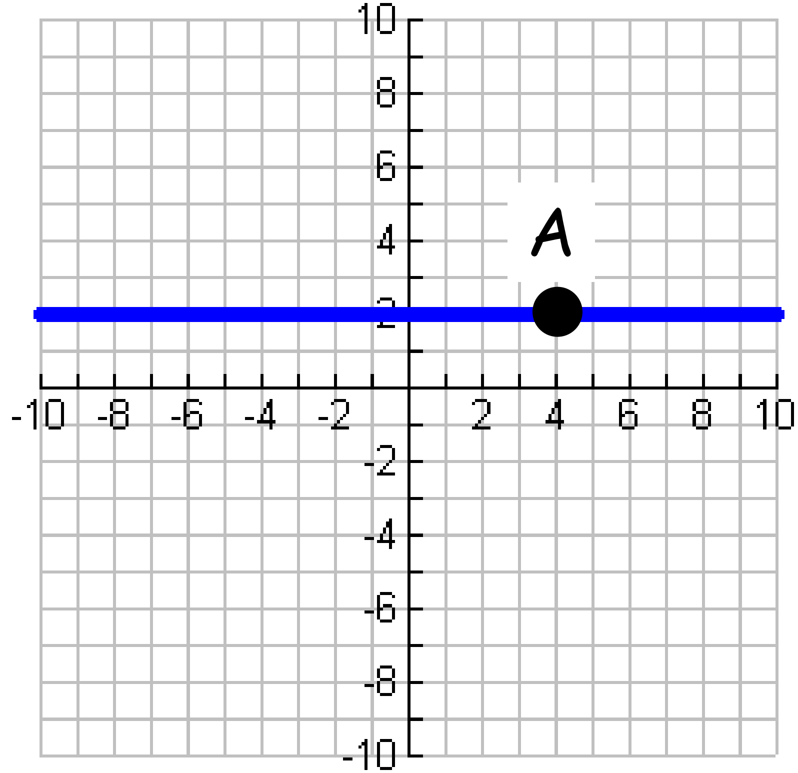
The change of the dependent variable over the change of the independent variable.

What is the slope at point A?

$$y = 2$$

$$\frac{dy}{dx} = 0$$

Slope = 0



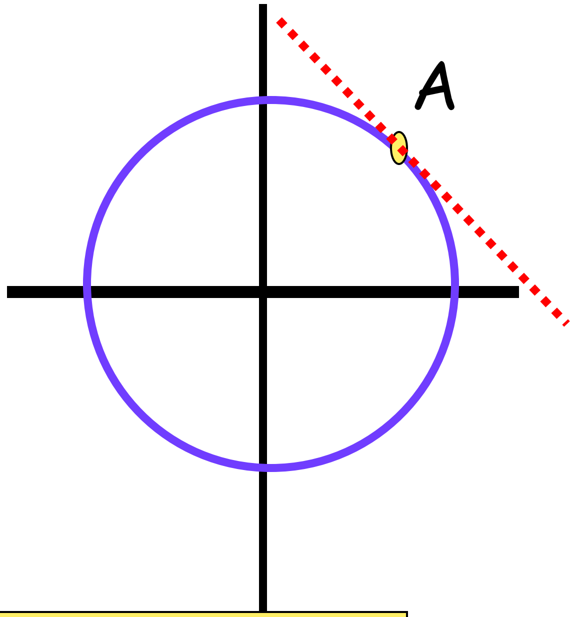
Now, let's look at a polar graph…

The equation is $r = 2$

What is the slope at point *A*?

Slope is DEFINITELY $\neq 0$

But $\dfrac{dr}{d\theta} = 0$



What's going on?

This means $\frac{dr}{d\theta}$ does not find the slope of the curve!

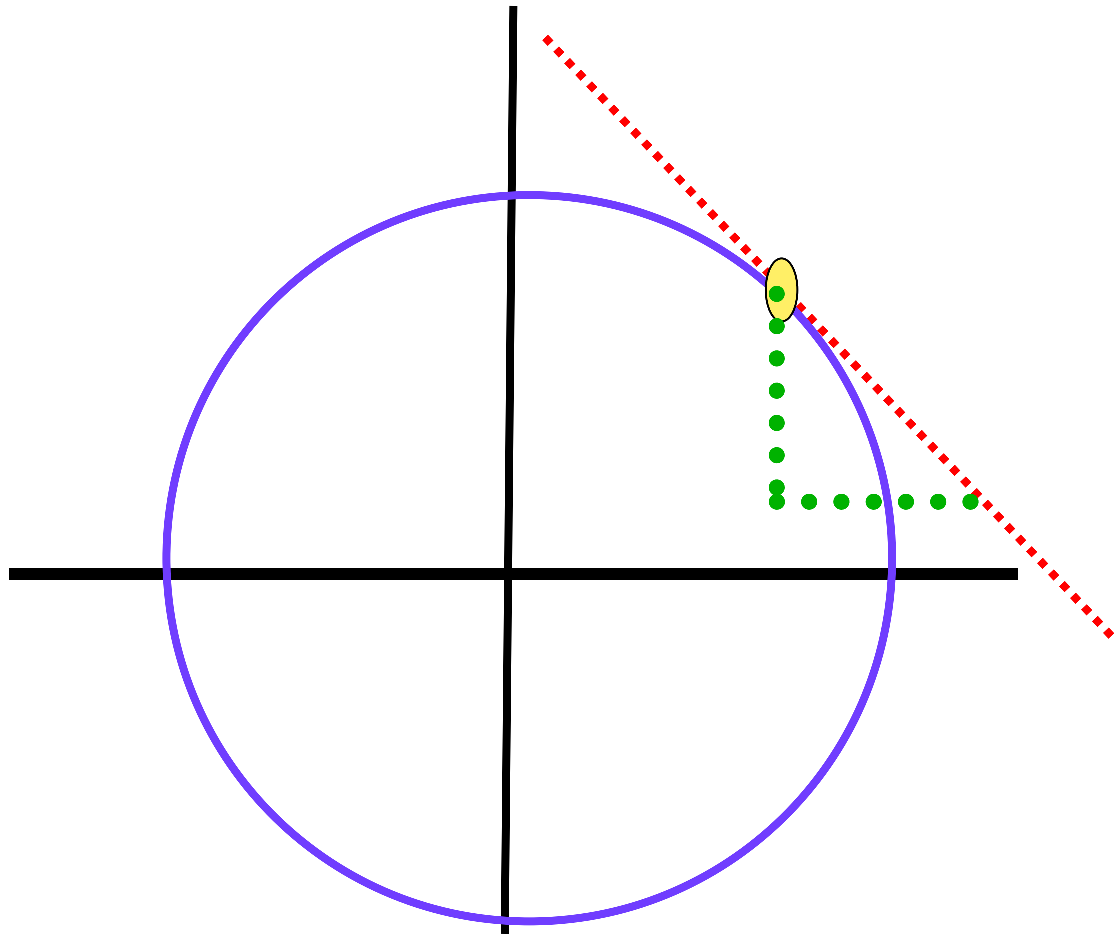
So what is $\frac{dr}{d\theta}$

The change in r with respect to theta.

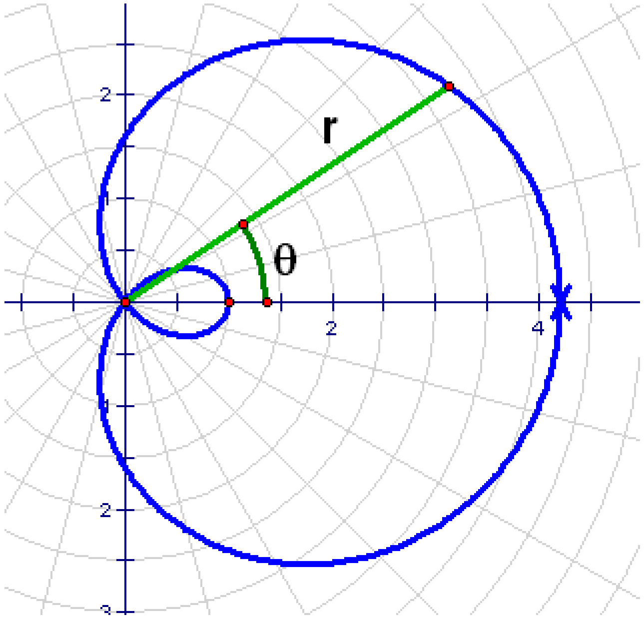
And in our circle, r did not change, which explains the 0.

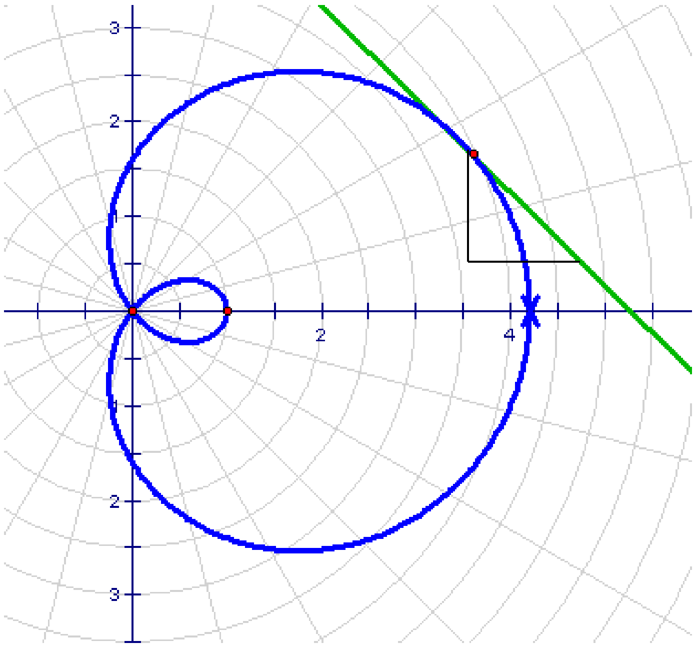
But what if we want the slope of the Polar curve?

What we really need is the dy/dx…



Compare and Contrast





$$\frac{dr}{d\theta} =$$ Rate of change in radius as theta changes.

$$\frac{dy}{dx} =$$ Slope of the tangent

But, we only have r's and theta's. What do we do?

Use conversion equations and write:

$$x = r\cos\theta = f(\theta)\ \cos\theta$$

$$y = r\sin\theta = f(\theta)\ \sin\theta$$

r is a function of θ

$$x = r\cos\theta = f(\theta)\cos\theta$$

$$y = r\sin\theta = f(\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} =$$

Let's go back to the original example…r = 2

It's clear that $\frac{dr}{d\theta} = 0$

Now, let's find the slope of the polar curve. REMEMBER this is dy/dx:

$$\frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

EXAMPLE #2

Find the slope of $r = 3(1-\cos\theta)$ at $\theta = \frac{\pi}{2}$

$$\left.\frac{dr}{d\theta}\right|_{\theta=0} = \_\_\_\_\_\_\_\_$$

$$r(0) = \_\_\_\_\_\_$$

$$\sin(0) = \_\_\_\_\_\_$$

$$\cos(0) = \_\_\_\_\_\_$$

Now . . .

Write the equation of the tangent line to $r = 3(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$

We need the slope at this location. Got it!

$$\frac{dy}{dx} =$$

And we need to know the coordinates of the point of tangency . . . .

Point of Tangency . . . .

$$r = 3(1 - \cos\theta) \quad \text{at} \quad \theta = \frac{\pi}{2}$$

REMEMBER the slope of the curve is dy/dx.

Which means the we need the Cartesian coordinates for our point of tangency.

BUT first we will find the polar coordinates

Polar: $\left(r\left(\frac{\pi}{2}\right), \frac{\pi}{2} \right) = \left(\_\_\_\_, \frac{\pi}{2} \right)$

NOW, convert $\left(3, \frac{\pi}{2}\right)$ to Cartesian

Remember, $x = r\cos\theta \quad and \quad y = r\sin\theta$

Now you can write the <u>equation</u> of the tangent line

Slope of Tangent line: $\dfrac{dy}{dx} = -1$

Point of Tangency: (0,3)

Next Example Problem

Find the lines tangent to the following rose curve at the pole:

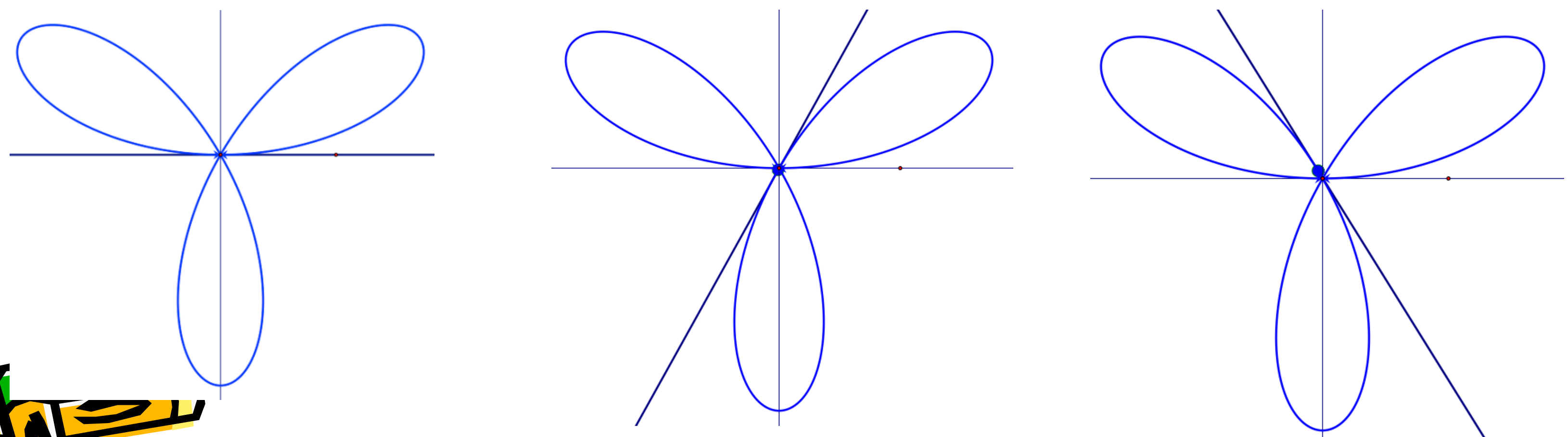
$$r = f(\theta) = 2\sin 3\theta$$

Remember the domain is

$0 \leq \theta \leq \pi$ because it is an "odd" rose curve.

Find the tangents lines to the curve at the pole.

$$r = 2\sin(3\theta), \quad 0 \le \theta \le \pi$$



It looks like we have 3 tangents.

First:

Since we are interested in tangent lines at the pole, we need to know the theta values that will make *r* =0.

$$0 = 2\sin 3\theta$$

BE CAREFUL. ☺

Second:

Since we are interested in tangent lines at the pole, the Cartesian coordinates for the point of tangency will always be (0,0).

Third:

Now for the slope at the pole . . . .

Slopes of tangent lines at the pole . . .

Since *r=0* at the pole

$$\frac{\frac{dr}{d\theta}\sin\theta + \cancel{r\cos\theta}}{\frac{dr}{d\theta}\cos\theta - \cancel{r\sin\theta}} =$$

Now, find the value of the slope (dy/dx) at the theta values from our previous step. And write the equation of the tangent line.

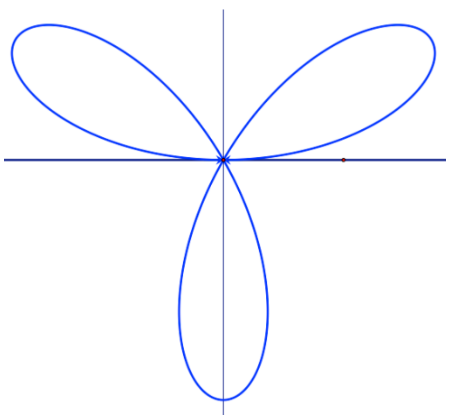
$\theta - value$	$\tan\theta$	Equation of Tangent Line
0		
$\pi/3$		
$2\pi/3$		
π		

Find the tangent lines to the curve at the pole.

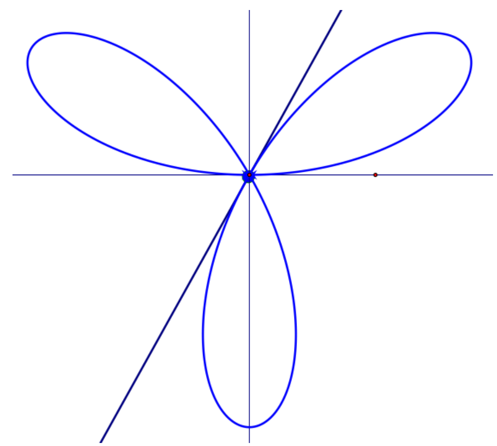
$$r = 2\sin(3\theta), \quad 0 \le \theta \le \pi$$

$$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

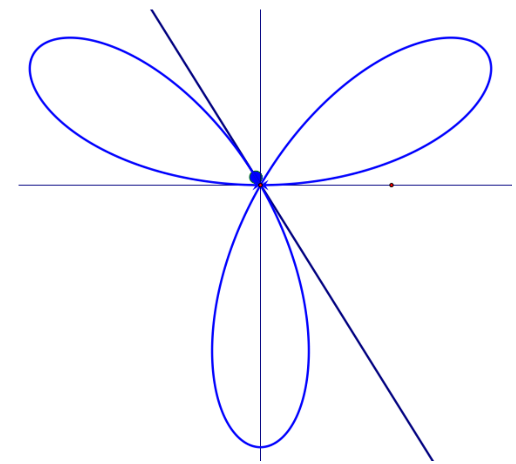
$$\frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$



$$\frac{dy}{dx} = \tan 0 = 0$$



$$\frac{dy}{dx} = \tan\frac{\pi}{3} = \sqrt{3}$$



$$\frac{dy}{dx} = \tan\frac{2\pi}{3} = -\sqrt{3}$$

Find the tangents lines to the curve at the pole.

$$r = 2\sin\left(3\theta\right), \quad 0 \le \theta \le \pi$$

$$\frac{dy}{dx} = \tan 0 = 0 \qquad \frac{dy}{dx} = \tan\frac{\pi}{3} = \sqrt{3} \qquad \frac{dy}{dx} = \tan\frac{2\pi}{3} = -\sqrt{3}$$

Straight lines through the pole are easy.

$$y = 0$$
$$y = \sqrt{3}x$$
$$y = -\sqrt{3}x$$

They are even easier in polar.

$$\theta = 0$$
$$\theta = \frac{\pi}{3}$$
$$\theta = \frac{2\pi}{3}$$

Summary

To find tangents at the pole:

- For polar equations, just find θ's
- For rectangular equations, dy/dx = tan(θ), points = (0,0)

New type of problem . . .

$$\frac{dy}{dx}=\frac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}}=\frac{\dfrac{dr}{d\theta}\sin\theta+r\cos\theta}{\dfrac{dr}{d\theta}\cos\theta-r\sin\theta}$$

To find horizontal tangents, the numerator of the slope expression must equal zero.

$$\frac{dy}{d\theta}=0$$

To find vertical tangents, the denominator of the slope expression must equal zero.

$$\frac{dx}{d\theta}=0$$

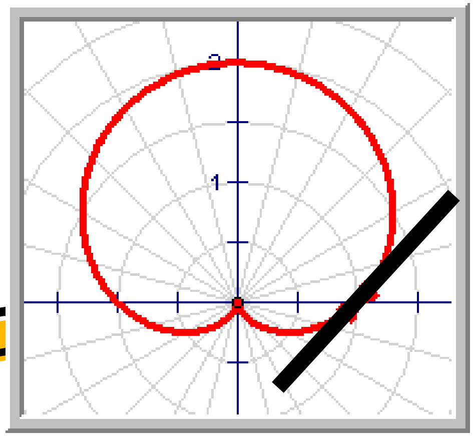
Example Problem . . .

For the curve, $r = 1 + \sin\theta$

A) Find the EQUATION for the slopes of the tangent lines

B) Find where there are horizontal tangents

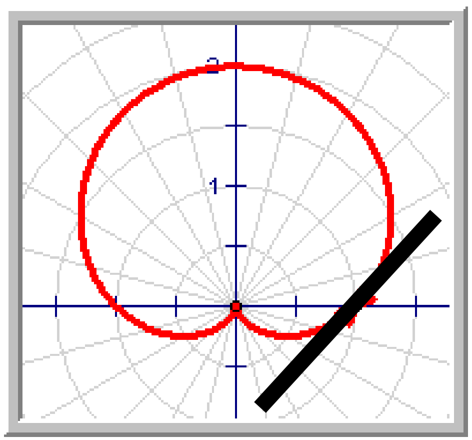
C) Find the slope when $\theta = 0$



Example Problem . . .

For the curve, $r = 1 + \sin\theta$

A) Find the EQUATION for the slopes of the tangent lines



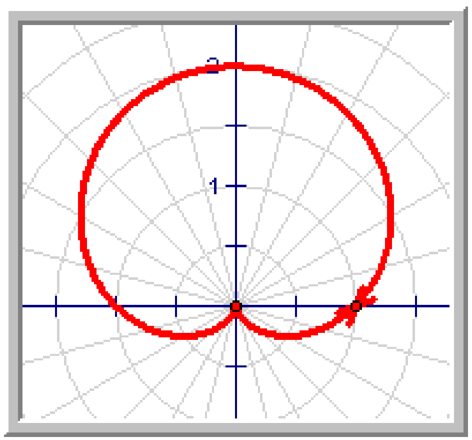
$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

A) $$\frac{dy}{dx} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$$

B) Find where there are horizontal tangents

$$\frac{dy}{dx} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$$

$$\frac{dy}{d\theta} = 0 \quad \Rightarrow \cos\theta + 2\cos\theta\sin\theta = 0 \quad \Rightarrow$$



$$\cos\theta \;\; 1 + 2\sin\theta \;\; = 0$$

$$\cos\theta = 0 \qquad\qquad 1 + 2\sin\theta = 0$$

$$\theta = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}} \qquad\qquad \theta = \frac{11\pi}{6}, \frac{7\pi}{6}$$

We discard $\frac{3\pi}{2}$ because it makes the derivative undefined

One More Thing . . .

Now that we know there is a horizontal tangent at $\frac{11\pi}{6}$ let's write the equation of the tangent line at this location.

Remember to write the equation of a line, we need a point and the slope.

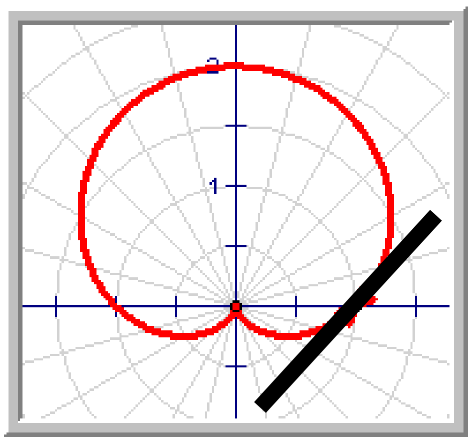
We know the slope is zero! Which makes this task easy to deal with once we know the point of tangency.

Example Problem . . .

For the curve, $r = 1 + \sin\theta$

C) Find the slope when $\theta = 0$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$



$$\left.\frac{dr}{d\theta}\right|_{\theta=0} = \_\_\_\_\_\_\_\_$$

$$r(0) = \_\_\_\_\_\_$$

$$\sin(0) = \_\_\_\_\_\_$$

$$\cos(0) = \_\_\_\_\_\_$$

Using Calculator to analyze graphs

TRACE

FORMAT Parameters
- PolarGC

WINDOW Parameters
- θMin, θMax
- θStep

Finding Polar intersections

DRAW Menu
- Horizontal and Vertical Tangent

DRAW

Set $r = 1 + 2\sin\theta$

Set $\theta Min = 0$

$\theta Max = 2\pi$

$\theta Step = \pi / 24$

DRAW Horizontal and Vertical can help visualize and locate tangents.

Option 1:ClrDraw to clear lines

Bring up DRAW menu (2nd PRGM)

Select 3:Horizontal

Use up/down arrows

Press ENTER to fix line

Draw

DRAW Tangent can show tangent at any point and its dy/dx value.

Select 5:Tangent

Use right/left arrows to move around graph to any point

Press ENTER to draw tangent line