BC Calculus

Unit 5 Day 7
Alternating Series

Today's Agenda

New Material:
Convergence of Alternating Series

HW Questions/Extra Practice

Quiz:
Direct and Limit Comparison Tests

HW Answers (Spring 2011)

- Extra Practice Handout

#6 DV by test for DV
#9 CV by direct comparison
#11 DV by test for DV
#13 DV by integral test
#32 B
#33 A
#34 D
#35 DV
#36 CV
#37 CV to 20
#38
#39 E

Alternating Series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$$

- Terms are alternately positive and negative.
  - Normally have a (-1) raised to a power
  - Ex:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} ...$$

The Alternating Series Test

An alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ CONVERGES if

$a)\ b_n > 0$
$(necessary\ \ for\ terms\ to\ alternate\ in\ sign)$

AND

$b)\quad b_n > b_{n+1} \quad for \quad all \quad n$

AND

$c)\quad \lim_{n\to\infty} b_n = 0$

Understanding each part of the Alternating Series Test

$a)\ b_n > 0$
$(necessary\ \ for\ terms\ to\ alternate\ in\ sign)$

Each of the b_n's is positive

$b)\quad b_n > b_{n+1} \quad for \quad all \quad n$

Each b_n is greater than its succeeding b_{n+1}

$c)\quad \lim_{n\to\infty} b_n = 0$

The b_n's are tending to zero as n approaches infinity.

Does the alternating Harmonic Series converge?

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots \quad = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$$

$a)\ b_n > 0$
$(necessary\ \ for\ terms\ to\ alternate\ in\ sign)$ Yes

$b)\quad b_n > b_{n+1} \quad for \quad all \quad n$ Yes

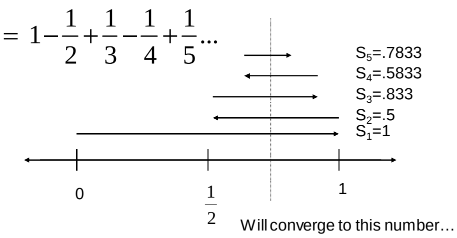
$c)\quad \lim_{n\to\infty} b_n = 0$ Yes

Therefore, CONVERGENT

Let's see why…

Since the absolute value of the terms decreases to 0, the partial sums bounce back and forth around some number…

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} \quad = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\ldots$$



You can probably imagine why this would not work if terms INCREASED in absolute value!

Another example…

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$$

$a)\ b_n > 0$ (necessary for terms to alternate in sign) — Yes

$b)\ b_n > b_{n+1}\quad for\quad all\quad n$ — Yes, 4n will grow faster than

$c)\ \lim_{n\to\infty} b_n = 0$ — limit approaches 3/4

Therefore, DIVERGENT

Tonight's HW:
Packet p. 9 (odds)
Alternating Series

2.DV	3. CV	4. CV
5. CV	6. CV	7. DV
8.CV	9. CV	10. DV
11. CV	12. CV	13. DV
14. CV	15. CV	16. CV
17. CV	18. DV	19. DV
20. DV		

Estimating what an alternating series converges to . . . .

- Done by calculating a partial sum.
  - Accuracy depends on how many terms are included in the partial sum
    - Difference between the actual sum and the partial sum is the error.

The size of the error is smaller than

$$b_{n+1}$$

Which is the absolute value of the first "neglected" term.

Alternating Series Estimation Theorem

If $s = \sum (-1)^{n-1} b_n$ is the sum of a convergent alternating series then

$$|R_n| = |s - s_n| \le b_{n+1}$$

Ex) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ correct to 3 decimal places

Is it convergent?

Now, where is that neglected term?

$\frac{(-1)^n}{n!}$

n		s_n
0	1	1
1	-1	0
2	$\frac{1}{2}$	$\frac{1}{2}$
3	$-\frac{1}{6}$	$.3333$
4	$\frac{1}{24}$	$.375$
5	$-\frac{1}{120} = -.008$	$.3666$
6	$\frac{1}{720} = .001$	$.368055$
7	$-\frac{1}{5040} = -.0002$	

$So,\ b_7 = \frac{1}{5040} < .0002$

$and\ s_6 \approx .368056$

By Alt. Series Estimation Theorem

$|s - s_6| \le b_7 < .0002$

Which does not affect the 3rd decimal place.

$\therefore s \approx .368$