$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

Day 6

$$P = \frac{M}{1 + Ae^{-kt}}$$

$$y = \frac{12}{1 + \frac{7}{5}e^{-t}}$$

1. Suppose that the growth of a population $y = y(t)$ is given by the logistic equation $y = \dfrac{60}{5 + 7e^{-t}}$

   a) What is the population at time t=0? $\frac{60}{5+7} = \frac{60}{12} = \boxed{5}$

   b) What is the carrying capacity L? (Hint: It is not 60. Carrying capacity = $\lim_{t\to\infty}$ ...)

   $\lim_{t\to\infty} \frac{60}{5+7e^{-t}} = \frac{60}{5+7(0)} = \frac{60}{5} = \boxed{12}$

   c) What is the constant k?

   $-1 = -k \rightarrow \boxed{k=1}$

   d) When does the population reach half of the carrying capacity?

   $6 = \frac{60}{5+7e^{-t}} \rightarrow \boxed{t \approx 0.3365}$

   e) Write an initial value differential equation problem whose answer would above given logistic equation. $\boxed{\frac{dy}{dt} = y\left(1 - \frac{y}{12}\right)}$

2. Suppose that the growth of a population $y = y(t)$ is given by the logistic equation

   $$y = \frac{1000}{1 + 999e^{-0.9t}}$$

   $\frac{1000}{1+999} = \frac{1000}{1000} = \boxed{1}$

   $750 = \frac{1000}{1+999e^{-0.9t}}$ $\boxed{t \approx 8.89}$

   a) What is the population at time t=0?
   b) What is the carrying capacity L? (See hint for 1b) $\leftarrow \lim_{t\to\infty} y = \boxed{1000}$
   c) What is the constant k? $-0.9 = -k \rightarrow \boxed{k = 0.9}$
   d) When does the population reach 75% of the carrying capacity? 75% of 1000 = 750
   e) Write an initial value differential equation problem whose answer would above given logistic equation. $\boxed{\frac{dy}{dt} = 0.9P\left(1 - P/1000\right)}$

3. Suppose that a population y(t) grows in accordance with the logistic model $\frac{dy}{dt} = 10(1 - 0.1y)y$

   $\frac{dy}{dt} = 10y\left(1 - \frac{y}{10}\right)$

   a) What is the carrying capacity? (Hint: When a logistic function approaches capacity, the rate slows down and approaches zero. For what y-values does $\frac{dy}{dt} = 0$) $\boxed{M=10}$
   b) What is the value of k? $\boxed{k=10}$
   c) For what value of y is the population growing most rapidly? (Hint: You find the maximum of any function by finding its derivative and critical points. The derivative of $\frac{dy}{dt}$ is $\frac{d^2y}{dt^2}$. In other words, find the 2nd derivative. Where it equals zero is where $\frac{dy}{dt}$ has its maximum and minimums.)

   $\frac{dy}{dt} = 10y - y^2$

   $\frac{d^2y}{dt^2}$ $10 - 2y$ $\quad 0 = 10 - 2y$, $2y = 10$ $\rightarrow y = 5$

   y'': $+$ | $-$; y': 0 $\nearrow$ 5 $\searrow$

   $\boxed{@\ y=5}$

4. Suppose that a population y(t) grows in accordance with the logistic model $\frac{dy}{dt} = 50y - 0.001y^2$

   $= 50y\left(1 - \frac{y}{50000}\right)$

   a) What is the carrying capacity? (See hint for 3a) $\boxed{M=50000}$
   b) What is the value of k? $\boxed{k=50}$
   c) For what value of y is the population growing most rapidly? (See hint for 3c)

   $\frac{d^2y}{dt^2} = 50 - 0.002y$

   $0 = 50 - 0.002y$

   12

   y'': $+$ | $-$; y': 0 $\nearrow$ 25000

   $\boxed{@\ y = 25000}$