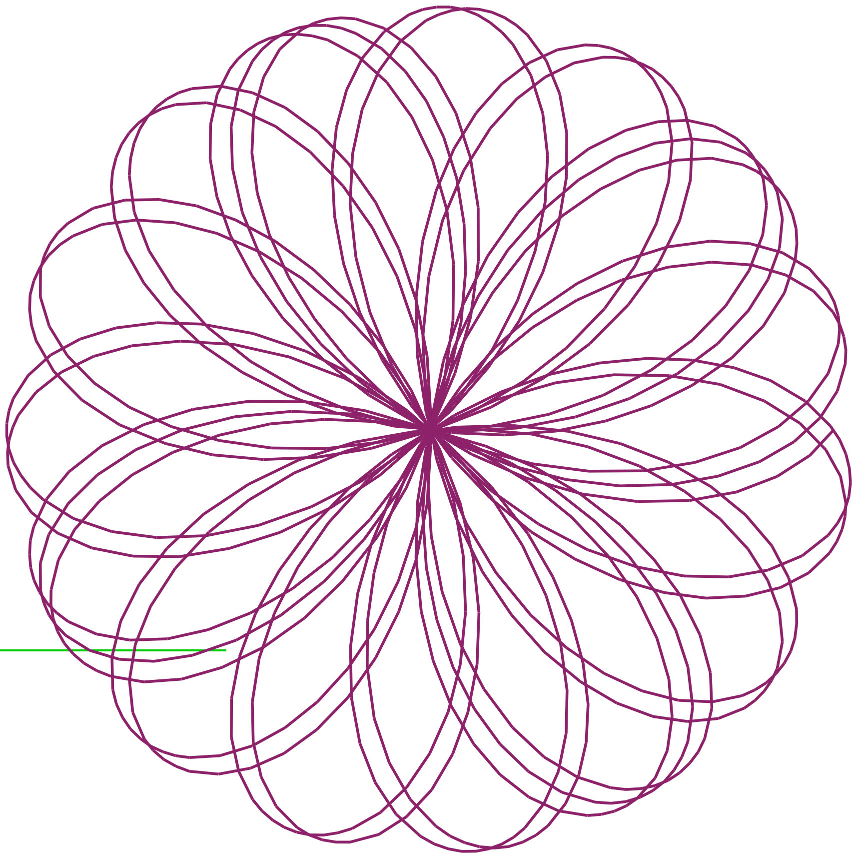
BC Calculus

Unit 1 Day 6



$$r = 2\sin\ 2.15\theta$$

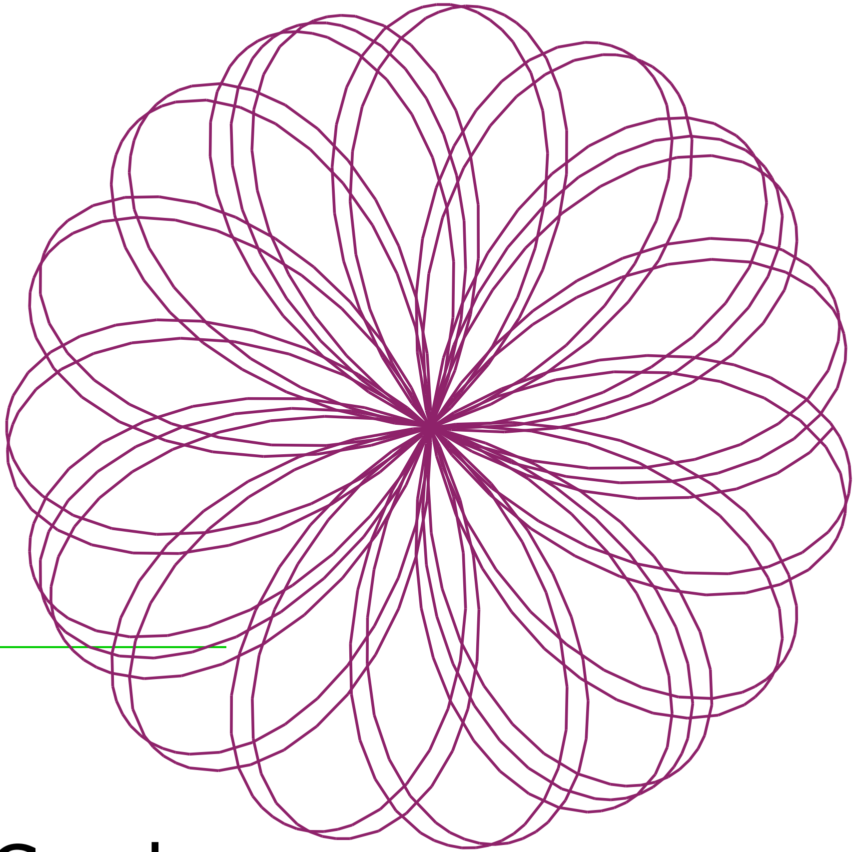
$$0 \le \theta \le 16\pi$$

Arrival Instructions

- Take out your homework
- Compare answers with people around you and determine which questions need to be addressed as a class.

HW Questions

BC Calculus



NEW TOPIC

Area **Between** Polar Graphs

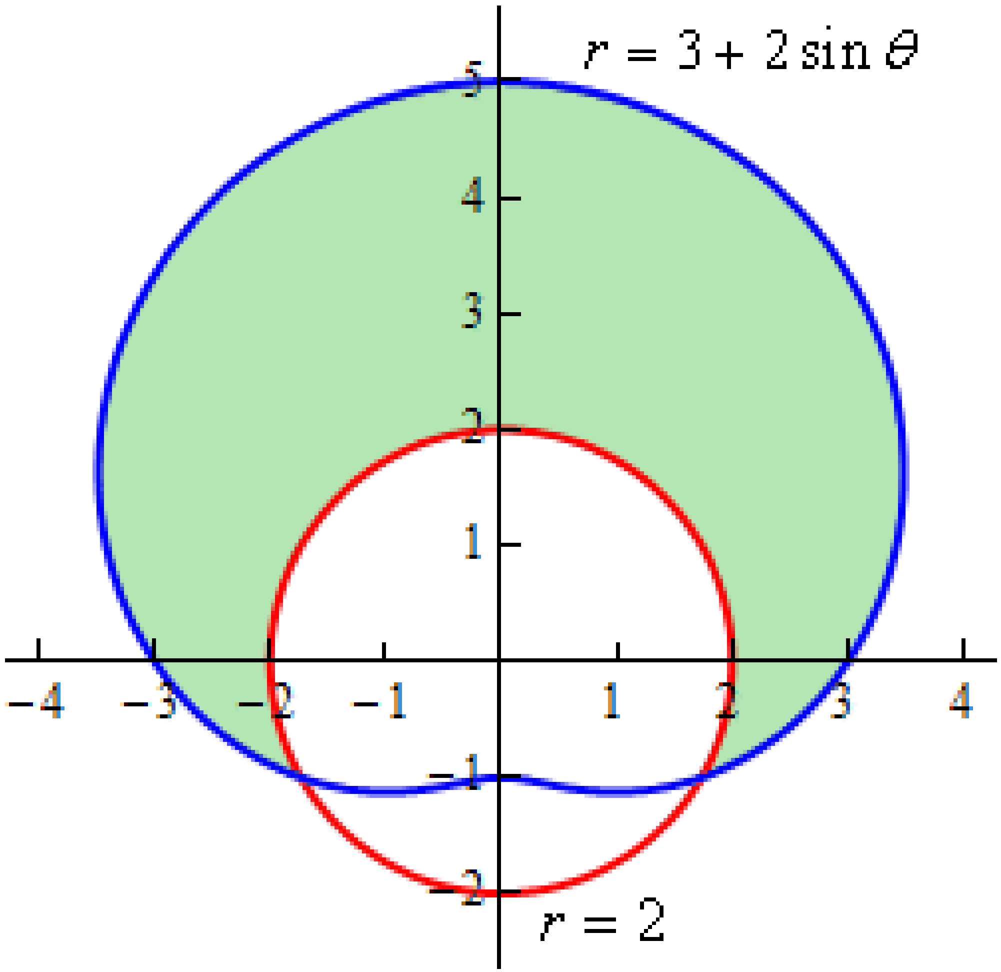
Day 6

Area **Between** Two Polar Curves

Determine the area that lies inside $r(\theta) = 3 + 2\sin\theta$ and outside $r(\theta) = 2$.

- First draw a quick sketch of the curves (without using your calculator)

Determine the area that lies inside $r(\theta) = 3 + 2\sin\theta$ and outside $r(\theta) = 2$.

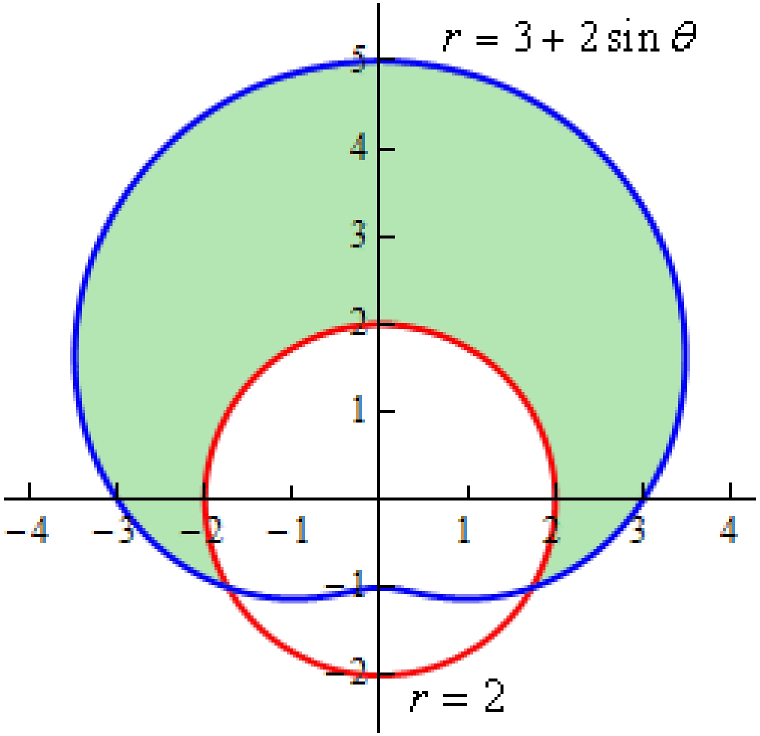


Find θ value of intersection points of the two curves:

Integratin Boundaries . . .

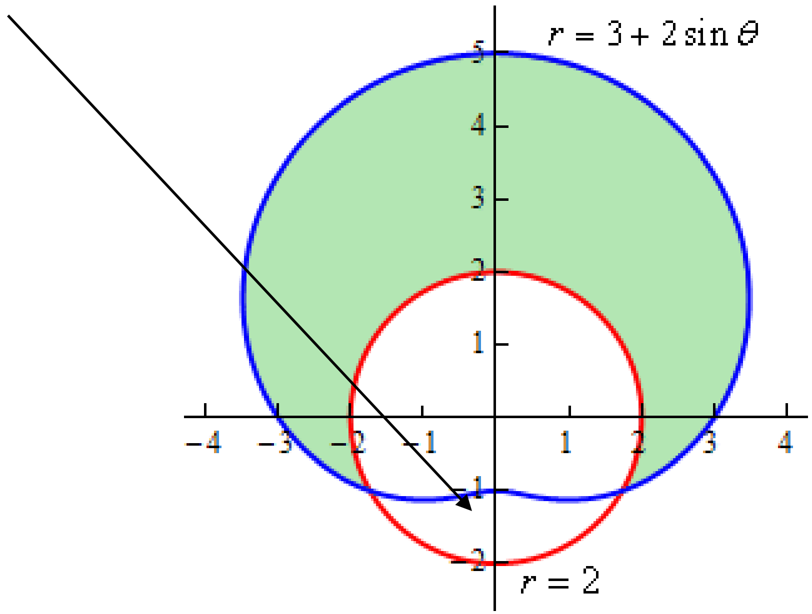
Our integration boundaries need to go from small to large

So, we can't use from $\frac{11\pi}{6}$ to $\frac{7\pi}{6}$

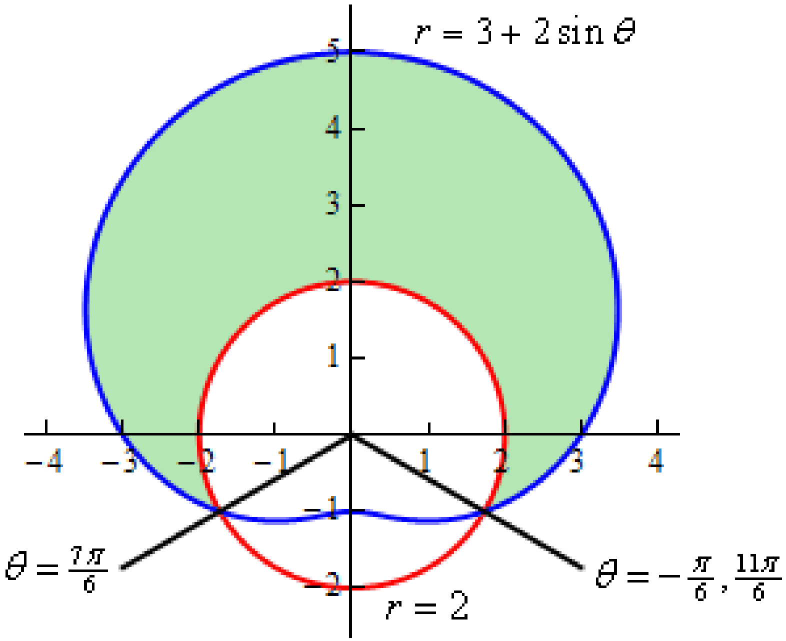


The issue . . . .

BUT, if we use $\frac{7\pi}{6}$ to $\frac{11\pi}{6}$ we will not be finding the shaded area but instead we would be finding the bottom most of the three regions.

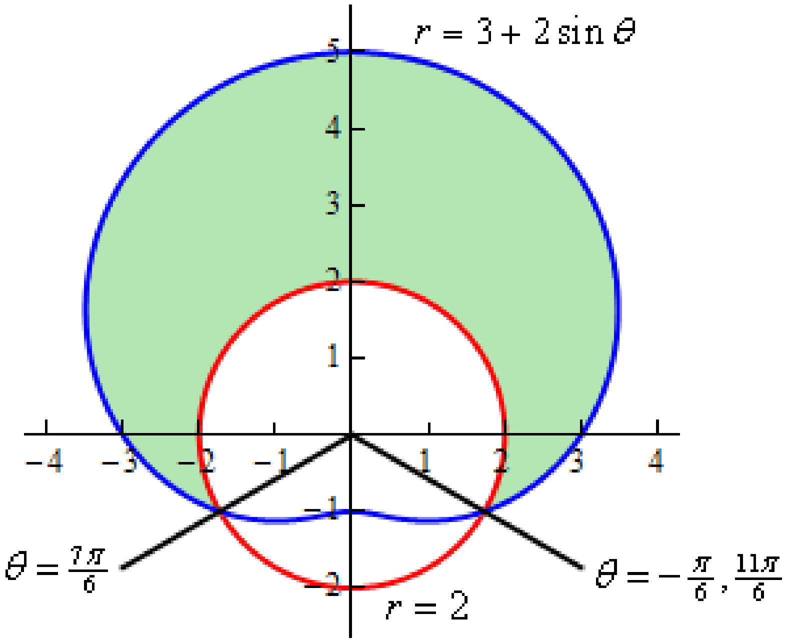


So . . . . . We need to adjust



Remember $\frac{11\pi}{6}$ and $-\frac{\pi}{6}$ are equivalent. And if we use these as the integration boundaries we will enclose the area that we're after.

So . . . . We need to adjust



Setup so far is . . .

$$\frac{1}{2}\int_{-\pi/6}^{7\pi/6} something$$

BUT what do we put in for the something?

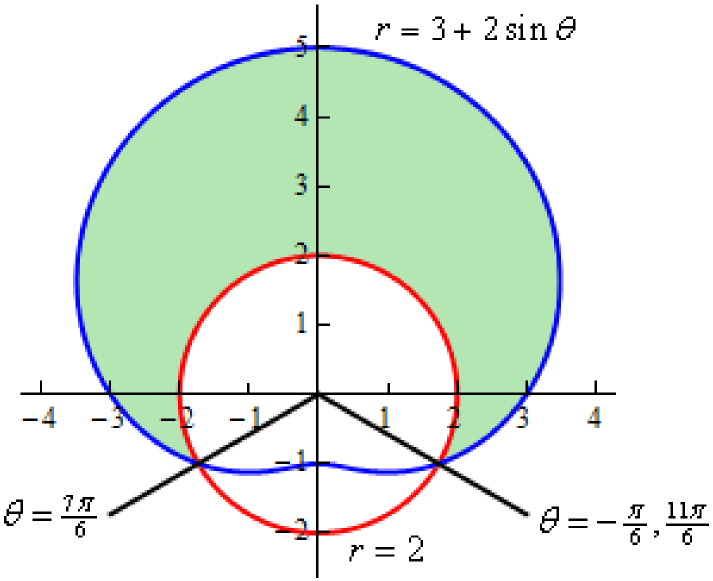
Let's explore . . . .

Here is the setup we will need . . . .

$$A = \frac{1}{2}\int_{\alpha}^{\beta} R^2 - r^2 d\theta$$

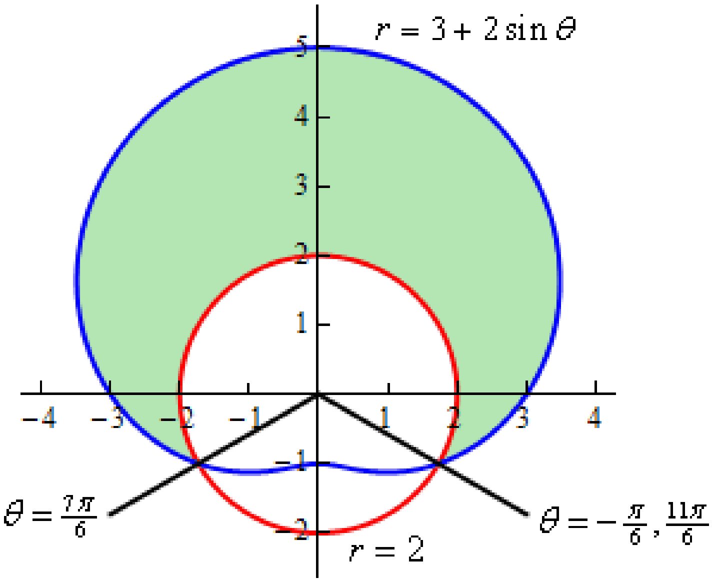
→

Finish the setup



$$\frac{1}{2}\int_{-\pi/6}^{7\pi/6} \left[\ 3+2\sin\theta\ ^{2}-(2)^{2}\right]d\theta$$

Now for the answer . . .



$$\int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left[\ 3+2\sin\theta\ ^2 - (2)^2\right] d\theta$$

$$= \int_{-\pi/6}^{7\pi/6} \frac{1}{2}\ 5+12\sin\theta+4\sin^2\theta\ \ d\theta$$

$$\int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left(5+12\sin\theta+4\left(\frac{1-\cos 2\theta}{2}\right)\right) d\theta$$

$$= \int_{-\pi/6}^{7\pi/6} \frac{1}{2}\ 7+12\sin\theta-2\cos 2\theta\ \ d\theta = \frac{11\sqrt{3}}{2}+\frac{14\pi}{3} = 24.187$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Lets try another

- Find the area outside of $r = 2$ and inside of $r = 2 + 2\cos\theta$
- Start with a sketch of the curves (WITHOUT a calculator).
- Shade the region

Identify if we have a Big R and Little r scenario

How could we make use of symmetry?

Now let's use the calculator

Put BIG R in r_1 and put little r in r_2

Set up the integral—Use your calculator

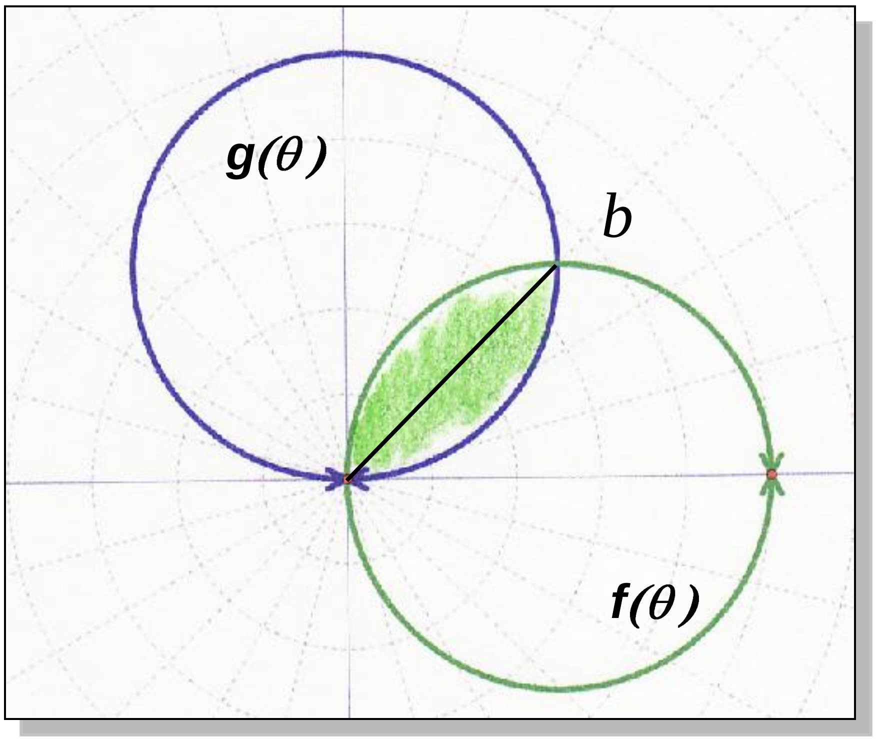
$$2\left[\frac{1}{2}\int_0^{\pi/2}(r_1)^2-(r_2)^2\,d\theta\right]$$

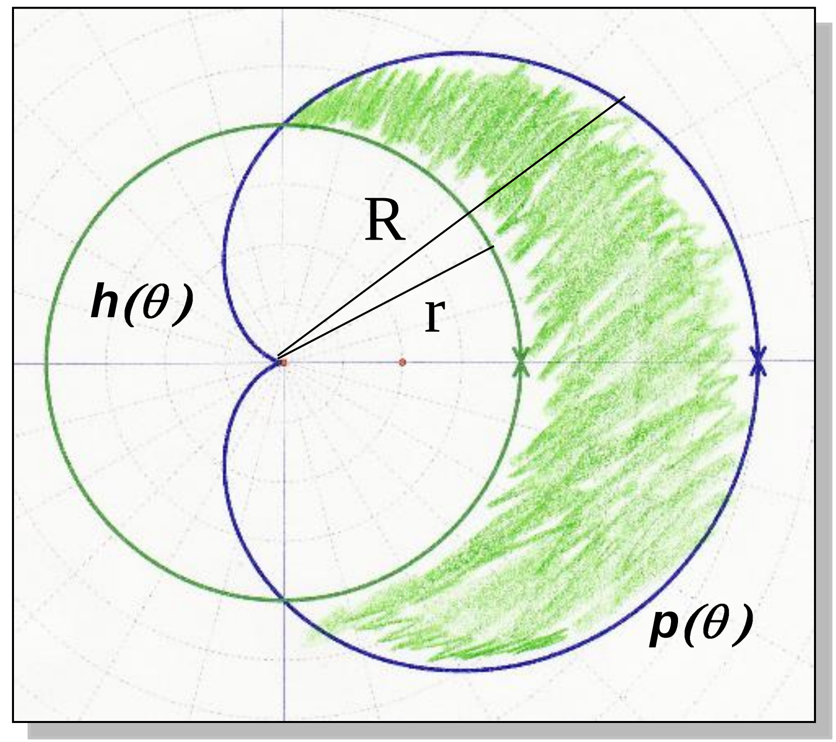
Questions??

- Find the area outside of $r = 2$ and inside of $r = 2 + 2\cos\theta$

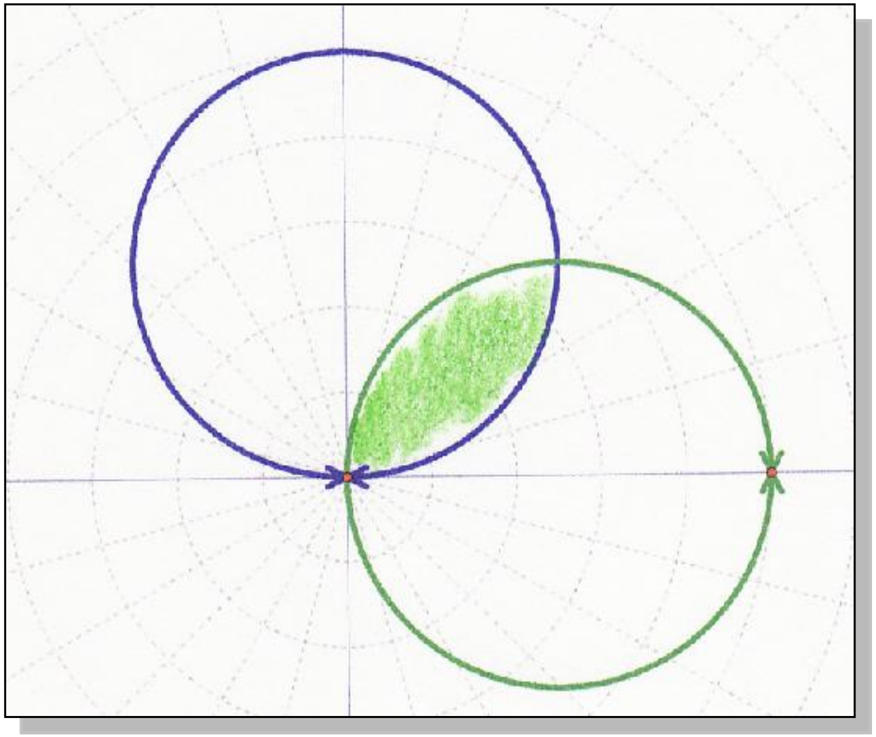
$$2\left[\frac{1}{2}\int_0^{\pi/2} (r_1)^2 - (r_2)^2 \, d\theta\right]$$

Compare and Contrast?





Find the area enclosed by



$$r(\theta) = \sin\theta$$

$$r(\theta) = \cos\theta$$