

Arrival:
Summarize the Series Tests So Far
(Use packet p. 1 if you want)

- * Geometric Series:
 DV or CV **AND** the CV value
- * Divergence Test:
 DV or Inconclusive
- * Telescoping:
 DV or CV **AND** the CV value
- * P-Series Test
 DV or CV **BUT** NOT the CV value
- * Integral Test
 DV or CV **BUT** NOT the CV value

Coming UP . . .

Comparison Tests
 Direct and Limit

Quick Review... CV or DV?

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ Geometric Series with $r = 1/2$

Converges

Specifically converges to $\frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 1$

Compare these series . . .

$\sum_{n=1}^{\infty} \frac{1}{2^n + 1}$ vs... $\sum_{n=1}^{\infty} \frac{1}{2^n}$

How do the terms in these series compare?

If a series has terms **SMALLER** than that of another **known CONVERGENT** series, then the smaller also **CONVERGES**.



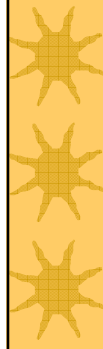
Use the Integral Test to answer . . .



Does $\sum_{n=1}^{\infty} \frac{5}{2n}$ converge or diverge?



Now a comparison . . .



Pulling out the constant,

$$\sum_{n=1}^{\infty} \frac{5}{2n} = \frac{5}{2} \sum_{n=1}^{\infty} \frac{1}{n}$$

How do the terms in this series compare to the Harmonic series?



Conclusion



$$\sum_{n=1}^{\infty} \frac{5}{2n} \text{ vs } \sum_{n=1}^{\infty} \frac{1}{n}$$

If we have a series whose terms are **LARGER** than the terms of a **known DIVERGENT** series, then the larger series will **DIVERGE** too!




DIRECT COMPARISON TEST (DCT)

for series with **POSITIVE TERMS**



If $\sum b_n$ converges and $a_n \leq b_n \forall n$, then $\sum a_n$ also converges.


If $\sum b_n$ diverges and $a_n \geq b_n \forall n$, then $\sum a_n$ also diverges.



DIRECT COMPARISON TEST
for series with POSITIVE TERMS

*Picking what to compare to . . .

- Geometric Series
- P-Series




Look for a SIMPLE series to compare to BUT be careful.

If the series is...	Might try Comparing to...
$\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$	$\sum_{n=1}^{\infty} \frac{n^2}{n^5}$ or $\sum_{n=1}^{\infty} \frac{1}{n^3}$

Disregard everything except the highest powers of n in the numerator and denominator when picking your series to compare to.

BUT for this problem WHAT is WRONG with even attempting to use the DCT?


Hint: What is the value of the first term?



Look for a SIMPLE series to compare to BUT be careful.

If the series is...	Compare to...
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$	$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Write out a few terms to see why this won't work.



Finally an example that DCT does work for

If the series is...	Compare to...
$\sum_{n=1}^{\infty} \frac{1}{3n^2 + 4n + 5}$	$\sum_{n=1}^{\infty} \frac{1}{n^2}$



Try this . . .

$$\sum_{n=0}^{\infty} \frac{3^n}{4^n + 5}$$

What would you pick as the comparison series?



New Test--LIMIT COMPARISON

Suppose $a_n > 0$ and $b_n > 0$ and for some finite value C , $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = C > 0$.

Then either $\sum a_n$ and $\sum b_n$ both converge or they both diverge.



LCT CONTINUED

*If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

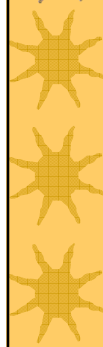
*If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.




Examples of Limit Comparisons

$$\text{ex1) } \sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$$

$$\text{ex2) } \sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$$




 **Examples of Limit Comparisons SOLUTIONS**

ex1) $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$

compare to: $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which is CV p-series since $p=2 > 1$

$$\lim_{n \rightarrow \infty} \frac{1/(3n^2 - 4n + 5)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2}{3n^2 - 4n + 5} = \frac{1}{3} > 0$$


\therefore both will CV

 **Examples of Limit Comparisons SOLUTIONS**

ex2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$

compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ which DV since p-series with $p=1/2 < 1$

\therefore both DV


 **YOU TRY the third one...**

ex3) $\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$

$$\lim_{n \rightarrow \infty} \frac{\frac{n^2 - 10}{4n^5 + n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{n^2 - 10}{4n^5 + n^3} \cdot \frac{n^3}{1} = \frac{1}{4}$$

Since this $\neq 0$, these series behave the same.

$\frac{1}{n^3}$ converges, thus $\frac{n^2 - 10}{4n^5 + n^3}$ converges too.

 **Packet p.8 (even)**
12.4 Direct and Limit Comparison

QUIZ TOMORROW
Telescoping
Integral Test
Direct and Limit Comparison Tests