Arrival:
Summarize the Series Tests So Far
(Use packet p. 1 if you want)

- Geometric Series:
  DV or CV AND the CV value
- Divergence Test:
  DV or Inconclusive
- Telescoping:
  DV or CV ***AND*** the CV value
- P-Series Test
  DV or CV ***BUT*** NOT the CV value
- Integral Test
  DV or CV ***BUT*** NOT the CV value

Coming UP . . .

Comparison Tests
Direct and Limit

Quick Review…CV or DV?

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$ Geometric Series with $r = 1/2$

Converges

Specifically converges to $\frac{a_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

Compare these series . . .

$$\sum_{n=1}^{\infty} \frac{1}{2^n+1} \quad \text{vs...} \quad \sum_{n=1}^{\infty} \frac{1}{2^n}$$

How do the terms in these series compare?

If a series has terms SMALLER than that of another known CONVERGENT series, than the smaller also CONVERGES.

Use the Integral Test to answer . . .

Does $\sum_{n=1}^{\infty} \frac{5}{2n}$ *converge or diverge*?

Now a comparison . . .

Pulling out the constant,

$$\sum_{n=1}^{\infty} \frac{5}{2n} = \frac{5}{2} \sum_{n=1}^{\infty} \frac{1}{n}$$

How do the terms in this series compare to the Harmonic series?

Conclusion . . . .

$$\sum_{n=1}^{\infty} \frac{5}{2n} \quad vs \quad \sum_{n=1}^{\infty} \frac{1}{n}$$

If we have a series whose terms are LARGER than the terms of a known DIVERGENT series, than the larger series will DIVERGE too!

DIRECT COMPARISON TEST (DCT)

for series with ***POSITIVE TERMS***

If $\sum b_n$ converges and $a_n \le b_n \ \forall$ n, then $\sum a_n$ also converges.

If $\sum b_n$ diverges and $a_n \ge b_n \ \forall$ n, then $\sum a_n$ also diverges.

DIRECT COMPARISON TEST

for series with POSITIVE TERMS

- Picking what to compare to . . .
  - Geometric Series
  - P-Series

Look for a SIMPLE series to compare to BUT be careful.

If the series is...

$$\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$$

Might try Comparing to...

$$\sum_{n=1}^{\infty} \frac{n^2}{n^5} \quad or \quad \sum_{n=1}^{\infty} \frac{1}{n^3}$$

Disregard everything except the highest powers of n in the numerator and denominator when picking your series to compare to.

BUT for this problem WHAT is WRONG with even attempting to use the DCT?

Hint: What is the value of the first term?

Look for a SIMPLE series to compare to BUT be careful.

If the series is...

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$$

Compare to...

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

Write out a few terms to see why this won't work.

Finally an example that DCT does work for . . . .

If the series is...

$$\sum_{n=1}^{\infty} \frac{1}{3n^2 + 4n + 5}$$

Compare to...

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Try this . . .

$$\sum_{n=0}^{\infty} \frac{3^n}{4^n + 5}$$

What would you pick as the comparison series?

New Test--LIMIT COMPARISON

Suppose $a_n > 0$ and $b_n > 0$ and for some finite value C, $\lim_{n\to\infty} \frac{a_n}{b_n} = C > 0$.

Then either $\sum a_n$ *and* $\sum b_n$ both converge or they both diverge.

LCT CONTINUED

- If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Examples of Limit Comparisons

*ex*1) $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$

*ex*2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n - 2}}$

Examples of Limit Comparisons SOLUTIONS

$ex1)\ \sum_{n=1}^{\infty}\frac{1}{3n^2-4n+5}$

compare to: $\sum_{n=1}^{\infty}\frac{1}{n^2}$, which is CV p-series since p=2>1

$$\lim_{n\to\infty}\frac{1/(3n^2-4n+5)}{1/n^2}=\lim_{n\to\infty}\frac{n^2}{3n^2-4n+5}=\frac{1}{3}>0$$

$\therefore$ both will CV

Examples of Limit Comparisons SOLUTIONS

$ex2)\ \sum_{n=1}^{\infty}\frac{1}{\sqrt{3n-2}}$

compare to $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ which DV since

p-series with $p=1/2<1$

$\therefore$ both DV

YOU TRY the third one…

$ex3)\sum_{n=1}^{\infty}\frac{n^2-10}{4n^5+n^3}$

$$\lim_{n\to\infty}\frac{\frac{n^2-10}{4n^5+n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{n^2-10}{4n^5+n^3}\bullet\frac{n^3}{1}=\frac{1}{4}$$

Since this # > 0, these series behave the same.

$\frac{1}{n^3}$ *converges, thus* $\frac{n^2-10}{4n^5+n^3}$ *converges too.*

Packet p.8 (even) 12.4 Direct and Limit Comparison

QUIZ TOMORROW

Telescoping

Integral Test

Direct and Limit Comparison Tests