3. According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of $32°C$ arrives at a mortuary where the temperature is kept at $10°C$. Then the differential equation satisfied by the temperature T of the corpse t hr later is

A

$\frac{dT}{dt} = -k(T_t - T_s)$

(A) $\frac{dT}{dt} = -k(T-10)$ (B) $\frac{dT}{dt} = k(T-32)$ (C) $\frac{dT}{dt} = 32e^{-kt}$

(D) $\frac{dT}{dt} = -kT(T-10)$ (E) $\frac{dT}{dt} = kT(T-32)$

4. If the corpse in the previous question cools to $27°C$ in 1 hour, then its temperature is given by the equation

C

$27-10=(32-10)e^{-kt}$

$\ln(17/22) = -k(1)$

$k = -\ln(17/22)$

$T_t - 10 = 22e^{-kt}$

$T_t = 10 + 22e^{-kt}$

(A) $T = 22e^{0.205t}$ (B) $T = 10e^{1.163t}$ (C) $T = 10 + 22e^{-0.258t}$

$T_0 = 32$

$T_s = 10$

$T_{60} = 27$

(D) $T = 32e^{-0.169t}$ (E) $T = 32 - 10e^{-0.093t}$

Free Response

5. According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. If a roast at room temperature $68°F$ is put into a $20°F$ freezer, and if, after 2 hours, the temperature of the roast is $40°F$: $T_t - T_s = (T_0 - T_s)e^{-kt}$

$T_0 = 68$

$T_s = 20$

$T_2 = 40$

(a) What is its temperature after 5 hours?

$40-20=(68-20)e^{-2k}$

$\ln(20/48) = -2k$

$k = \frac{\ln(5/12)}{-2} \approx 0.4377$

$T_5 = 48e^{-5k} + 20$

$= \boxed{25.379°F}$

(b) How long will it take for the temperature of the roast to fall to $21°F$?

$21 = 48e^{-kt} + 20$

$\ln(\frac{1}{48}) = -kt$

$t = \frac{\ln(1/48)}{-k} \approx \boxed{8.84 \text{ hrs}}$