AP Calculus BC

Unit 4 Day 5

Newton's Law of Cooling

Agenda

- HW Questions
- Newton's Law of Cooling—Differential Equation to Formula
- CSI Lab

Newton's Law of Cooling

Newton's Law:

Temperature of an object decreases at a rate proportional to the difference between its current temperature and surrounding air.

Differential Equation: $\frac{dT}{dt} = -k(T - T_s)$

where T is the current temperature and T_s is the surrounding temperature

Converting to Exponential Equation

Differential Equation: $\frac{dT}{dt} = -k(T - T_s)$

where T is the current temperature and T_s is the surrounding temperature

Separate and Integrate!

- Must assume that T_s is constant
- Remember T is the variable that we are solving for
- At $t=0$ the temperature is T_0

The result would be . . .

Exponential Equation:

$$T - T_s = (T_o - T_s)e^{-kt}$$

Let's look at the Separating and Integrating

$$\frac{dT}{dt} = -k(T - T_s)$$

$$\int \frac{dT}{T - T_s} = \int -k\ dt$$

$$\ln|T - T_s| = -kt + C$$

$$e^{\ln|T - T_s|} = e^{-kt+C}$$

Separating and Integrating

$$e^{\ln|T - T_s|} = e^{-kt+C}$$

$$|T - T_s| = Ae^{-kt}$$

$$T - T_s = Ae^{-kt}$$

This value will always be positive so we can drop the absolute value.

Solve for the constant *A* given the temperature at $t=0$ is T_0

$$T_0 - T_s = A$$

$$T - T_s = (T_o - T_s)e^{-kt}$$

Newton's Law of Cooling

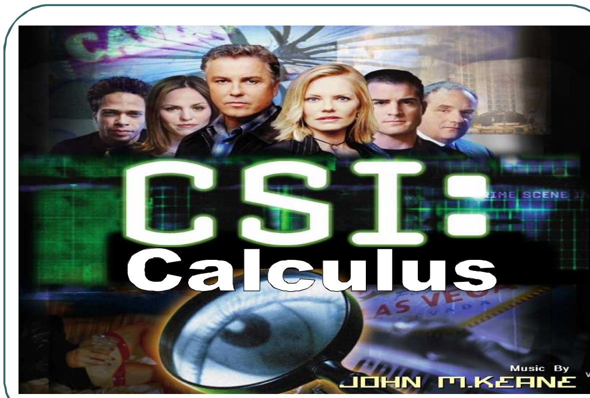
- Temperature of an object decreases at a rate proportional to the difference between its current temperature and surrounding air

Differential Equation:

$$\frac{dT}{dt} = -k(T - T_s)$$

Exponential Equation:

$$T - T_s = (T_o - T_s)e^{-kt}$$



CSI Example: Tom the Cat Case File

Facts:

- The body of Tom the Cat is found in a room that is 72˚F.
- When the SRMHS CSI team arrives, Tom the Cat's body temperature was 96˚F.
- His body temperature ½ hour later was 92˚F.
- Tom the Cat's body temperature when he was alive was 101˚F.

SRMHS CSI Team Collects some EXTRA information:

Facts (continued):

- An empty pill bottle was found at the scene along with this prescription:

Patient: Tom the Cat

Medicine: Prescription strength Cat Nip

Dosage directions:

Take 1 pill every *H* hours, where *H* is the absolute maximum value of $f(x) = x^3 - 3x^2 - 9x + 20$ on the closed interval [0,6].

How often did he need to take his medicine?

Your Mission as a SRMHS CSI Team Member is to determine how long Tom the Cat was dead when his body was found.

Remember the facts:

- The body of Tom the Cat is found in a room that is 72˚F.
- Body temperature upon arrival of the CSI GHHS team was 96˚F
- Body temperature ½ hour later was 92˚F
- Tom the Cat's body temperature when he was alive was 101˚F.

$$T_s = \_\_\_\_\_ \quad T_{normal} = \_\_\_\_\_\_$$

$$T_o = \_\_\_\_\_ \quad T = T_{30} = \_\_\_\_\_\_$$

Being a Detective

$$T_t - T_s = (T_o - T_s)e^{-kt}$$

First step: Find *k*

Being a Detective

$$T_t - T_s = (T_o - T_s)e^{-kt}$$

Second step:
Find *time* to get to temperature at time of discovery

What Happened to Tom the Cat

Tom was a pretty bright feline and knew to consider endpoints while looking for absolute extrema on a closed interval.

Thus, he found that (6,74) was the absolute maximum point of $f(x) = x^3 - 3x^2 - 9x + 20$.

However, Tom was very careless . . . .

What Happened to Tom the Cat

He was supposed to take his medicine every 74 hours, which is the function value ("y-coordinate") of the absolute maximum point. This is **the** "absolute maximum".

Instead he took his medicine every 6 hours and overdosed on prescription strength catnip. This is **when/where** the absolute maximum takes place.

The moral of the story:

Know what the question is asking for!!!

CSI: Calculus

Investigating Officers: ________________
____

Location of crime:

Victim:

Normal body temperature:

Body temperature on arrival:

Body temperature ____ minutes later:

Room temperature:

Calculations:

For Each CASE you examine.

How long had victim been dead when we arrived: (3 point decimal answer)

Backup for at least one case

Victim (*include name, not just type of animal*):

Start with $\frac{dT}{dt} = -k\left(T - T_s\right)$ *and show all necessary work to solve.*