u-Substitution and Inverse Trig Functions
Arrival

- Pick a group of 3 people. (There may be TWO groups of 2..)
- ONE member per group comes up to the front and picks a number from the Ms. Murphy.
- Select ONE piece of construction paper per group and write all names at the top of the paper.
Activity (packet pg. 7)

- Work on the problem that corresponds to the # you drew
- No Packet? Here are the problems . . .

1. $\int (3x + 1)^4 \, dx$

2. $\int xe^{-x^2} \, dx$

3. $\int x\sqrt{9 - x^2} \, dx$

4. $\int \frac{x - 2}{(x^2 - 4x + 3)^3} \, dx$

5. $\int 4(3x + 1)^3 \, dx$

6. $\int 4x(9 - x^2)^3 \, dx$

7. $\int 18x^2 \cos(3x^3) \, dx$

8. $\int 12x^3 \sin(3x^4) \, dx$

THEN… Show your work to Ms. Murphy! Once approved, you may hang up your paper in the back of the room.
When finished...

Grab a white board and solve for “y”

\[
\text{Solve } \frac{dy}{dt} = e^{y+2t}
\]

Hint: Use a property of exponents to separate the exponents
HW Questions?---TOMORROW!!

TOMORROW!!
QUIZ
Today’s Learning Outcomes

- Identify the “u” for the most effective u-substitution for integrating
- Find antiderivatives by substitution of variables
- Integrate inverse trig functions using u-substitution
- Evaluate definite integrals using u-substitution
Practice

\[ \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx \]

What should we pick for \( u \) ?

\[ u = \sin x \quad \text{OR} \quad u = \cos x \]

\[ du = \cos x \, dx \quad \text{or} \quad du = -\sin x \, dx \]

We would need \( \frac{1}{\cos x} \).

Not a good pick for \( u \).

\[ -\int \frac{1}{u} \, du \]

\[ = -\ln |u| + C = -\ln |\cos x| + C \]
U-Substitution—More About Picking \( u \)

Helpful Hints #1 and #2

1. Try to find which factor is close to being the derivative of the other

\[
\int (x^2 + 5)^3 (2x) \, dx = \int \frac{d}{dx} (x^2 + 5) \, dx = 2x \\
\text{therefore, } u = x^2 + 5
\]

2. The higher power is often \( u \)

\[
\int \sin^3 x \cos x \, dx
\]

3rd power \quad 1st power

therefore, \( u = \sin x \)
Helpful Hint # 3

3. Try letting the denominator be \( u \)

Check to see that the numerator is \( du \)

\[ \int \frac{\sin x}{\cos x} \, dx = \int \frac{\sin x}{\cos x} \, du = \int \frac{\sin x}{\cos x} \, dx \]

\[ u = \cos x \]

\[ du = -\sin x \, dx \]

\[ -du = \sin x \, dx \]
Helpful Hint # 4

\[ \int \frac{\ln^9 x}{x} \, dx \]

4. If \( \ln x \) is involved pick that for “\( u \)”
In class practice . . . .

Evaluate the following definite integrals.

1. \[ \int_{0}^{1} r \sqrt{1 - r^2} \, dr \]

2. \[ \int_{\pi/4}^{3\pi/4} \cot x \, dx \]
Here’s a tricky one:

\[ \int x \sqrt{5x + 2} \, dx \]

\[ \int x \sqrt{5x + 2} \, dx \]

Use original:

\[ u = 5x + 2 \]
\[ du = 5 \, dx \]
\[ \frac{1}{5} \, du = dx \]

\[ \int \frac{1}{5} (u - 2) u^2 \, \frac{1}{5} \, du \]

\[ \frac{1}{5} (u - 2) = x \]
\[
\int \frac{1}{5} (u - 2) u^2 \frac{1}{5} du
\]

\[
\frac{1}{25} \int (u - 2)u^2 du
\]

\[
\frac{1}{25} \int \left( u^2 - 2u^2 \right) du
\]

Solve, but don’t forget to substitute back in the x’s!
Final steps should be:

\[
\frac{1}{25} \int \left( u^2 - 2u^2 \right) du = \frac{1}{25} \left[ \frac{2}{5} u^2 - \frac{4}{3} u^2 \right] + C
\]

\[
= \frac{1}{25} \left[ \frac{2}{5} (5x + 2)^2 - \frac{4}{3} (5x + 2)^2 \right] + C
\]

\[
= \frac{1}{25} (5x + 2)^2 \left[ \frac{2}{5} (5x + 2) - \frac{4}{3} \right] + C
\]

Lovely!
u-Substitution with Inverse Trig Functions
Try to Solve using u-Substitution

\[ \int \frac{4}{4x^2 + 1} \, dx \]

\[ u = 4x^2 + 1 \]
\[ du = 8x \, dx \]

There is no extra \( x \) in the original to substitute for.
Enter Inverse Trig Functions

You need to remember the derivatives of the inverse trig functions:

\[
\frac{d \cos^{-1} u}{dx} =
\]

\[
\frac{d \sin^{-1} u}{dx} =
\]

\[
\frac{d \tan^{-1} u}{dx} =
\]

\[
\frac{d \csc^{-1} u}{dx} =
\]

\[
\frac{d \sec^{-1} u}{dx} =
\]

\[
\frac{d \cot^{-1} u}{dx} =
\]
Inverse Trig Integrals

\[ \int \frac{4}{4x^2 + 1} \, dx \]

What inverse trig function looks like the best fit?

Get the 1 in the bottom

Factor constants outside the integral

Write as (something)²

Use u-Substitution with inverse trig formula
More Practice!!!

2. \[ \int \frac{40}{x^2 + 25} \, dx \]

3. \[ \int \frac{dx}{\sqrt{4 - x^2}} \]

4. \[ \int \frac{dx}{\sqrt{1 - (x + 1)^2}} \]

5. \[ \int \frac{t}{\sqrt{1-t^4}} \, dt \]
More Practice Answers

2. \[ \int \frac{40}{x^2 + 25} \, dx = 8 \tan^{-1} \left( \frac{x}{5} \right) + C \]

3. \[ \int \frac{dx}{\sqrt{4 - x^2}} = \sin^{-1} \left( \frac{x}{2} \right) + C \]

4. \[ \int \frac{dx}{\sqrt{1 - (x+1)^2}} = \sin^{-1} (x+1) + C \]

5. \[ \int \frac{t}{\sqrt{1-t^4}} \, dt = \frac{1}{2} \sin^{-1} t^2 + C \]
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Thumbs up?
I would recommend you do the work on separate paper.