Unit 2 Day 5

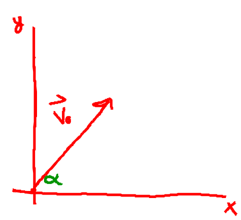
Ideal Projectile Motion

Homework Questions

"Ideal" projectile motion

- Projectile object moving in a vertical plane and the ONLY force acting on the projectile is gravity (downward force).
- Not considering more complicated problems involving air resistance, effects of earth's rotation etc.

The background . . . .



If an object is launched from the origin with an **initial velocity** of $\vec{\mathbf{v}}_0$ and **launch angle** of α,

Then $|v_0|$ is the initial speed (magnitude).

And

$$\vec{\mathbf{v}}_0 = (|v_0|\cos\alpha)\,\mathbf{i} + (|v_0|\sin\alpha)\,\mathbf{j}$$

Newton's 2nd Law of Motion

- Shows the relationship between **a force acting on an object**, the **mass of an object** and the **acceleration of an object**.

F*=*ma

Deriving the vector equation for the position of a projectile object . . . .

$$ma = m\left(\frac{d^2r}{dt^2}\right)$$

Acceleration is second derivative of position

$$m\left(\frac{d^2r}{dt^2}\right) = -mg\mathbf{j}$$

Gravity is "accelerating" our object in a downward direction

$$\left(\frac{d^2r}{dt^2}\right) = -g\mathbf{j}$$

$$\left(\frac{d^2r}{dt^2}\right) = -g\mathbf{j}$$

Separate and integrate.

$$\int d^2r = \int -g\mathbf{j}\, dt^2$$

$$\frac{dr}{dt} = (-gt)\mathbf{j} + C$$

Remember:

$$\frac{dr}{dt} = \vec{v}_0 \text{ at } t = 0$$

$$\vec{v}_0 = (-g0)\mathbf{j} + C$$

$$\therefore \vec{v}_0 = C$$

$$\frac{dr}{dt} = (-gt)\mathbf{j} + \vec{v}_0$$

Separate and integrate.

$$\int dr = \int (-gt)\mathbf{j} + \vec{v}_0\, dt$$

$$r = \left(-\frac{1}{2}gt^2\right)\mathbf{j} + \vec{v}_0 t + C$$

Initial position: $r(0) = \vec{r}_0$

$$\therefore C = \vec{r}_0$$

$$\mathbf{r} = \left(-\frac{1}{2}gt^2\right)\mathbf{j} + \vec{v}_0 t + \vec{r}_0$$

$$\mathbf{r} = \left(-\frac{1}{2}gt^2\right)\mathbf{j} + \left(|v_0|\cos\alpha\right)t\,\mathbf{i} + \left(|v_0|\sin\alpha\right)t\,\mathbf{j} + \vec{\mathbf{0}}$$

Often more simply written as:

$$r = (|v_0|\cos\alpha)t\mathbf{i} + \left((|v_0|\sin\alpha)t - \frac{1}{2}gt^2\right)\mathbf{j}$$

If the object is launched from a point (x_0,y_0) instead of the origin . . .

- Adjust the horizontal and vertical components

$$r = (x_0 + (v_0\cos\alpha)t)\mathbf{i} + \left(y_0 + (v_0\sin\alpha)t - \frac{1}{2}gt^2\right)\mathbf{j}$$

These should be in your notes

- **Position Vector Equation**:

  $$r = (x_0 + (v_0\cos\alpha)t)\mathbf{i} + \left(y_0 + (v_0\sin\alpha)t - \frac{1}{2}gt^2\right)\mathbf{j}$$

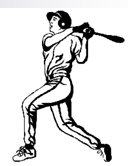
  Where g = gravity (9.8 m/s^2 or 32 ft/s^2)
- **Horizontal Position**:

  $$x = x_0 + (v_0\cos\alpha)t$$

- **Vertical Position**:

  $$y = \left(y_0 + (v_0\sin\alpha)t - \frac{1}{2}gt^2\right)$$

Example Problem:



- A baseball is hit from a height of 3 ft with an initial speed of 120 ft/s and an angle of 30 degrees above the horizontal (axis). Find a vector-valued function describing the position of the ball *t* seconds after it is hit.
- To be a home run, the ball must clear a wall that is 385 ft away and 6 ft tall. Determine if this hit is a home run.

Highest point reached ***when***. . .

- Vertical ***velocity*** component is zero

$$\frac{dy}{dt} = v_0 \sin\alpha - gt = 0$$

- Rate of change of vertical position is zero

The maximum height ***is*** . . .

- The vertical component of **position** at this *t* value

$$y_{\max} = y_0 + (v_0 \sin\alpha)(t_{highest}) - \frac{1}{2} g (t_{highest})^2$$

Example Problem:

Determine the **time to reach the maximum height** and the maximum height of a projectile fired at a height of 3 feet above the ground with an initial velocity of 900 feet per second and at an angle of 45 degrees above the horizontal.

WHEN does the projectile land (assuming launched from origin)

- Vertical component of position = 0

$$(v_0 \sin\alpha)t - \frac{1}{2}gt^2 = 0$$

Range (how far traveled in horizontal direction) . . .

- Horizontal Component of position at this *t* value

$$R = (v_0 \cos\alpha)\left(t_{ground}\right)$$

At what launch angle will an object obtain its maximum range?

$$(v_0 \sin\alpha)t - \frac{1}{2}gt^2 = 0$$

On Ground

$$R = (v_0 \cos\alpha)\left(\frac{2v_0 \sin\alpha}{g}\right) = \frac{(v_0)^2}{g}(2\cos\alpha \sin\alpha)$$

$$= \frac{(v_0)^2}{g}\sin 2\alpha$$

Horizontal position at the time hits the ground

At what launch angle will an object obtain its maximum range?

$$R = (v_0 \cos\alpha)\left(\frac{2v_0 \sin\alpha}{g}\right) = \frac{(v_0)^2}{g}(2\cos\alpha \sin\alpha)$$

$$= \frac{(v_0)^2}{g}\sin 2\alpha$$

Horizontal Range at the time until on ground value

$$\sin 2\alpha = 1$$

$$\therefore \alpha = \frac{\pi}{4}$$