Arrival Activity—Convert the following repeating decimal to a geometric series.

$$0.25252525...$$
$$= 0.25 + .0025 + .000025 + .00000025...$$
$$a = 0.25$$
$$r = 0.01$$
$$0.25252525... = \sum_{n=1}^{\infty} (0.25)(0.01)^{n-1}$$

Determine what this geometric series converges to.

$$0.25252525... = \sum_{n=1}^{\infty} (0.25)(0.01)^{n-1}$$
$$a = 0.25$$
$$r = 0.01$$
$$s = \frac{a}{(1-r)} = \frac{0.25}{(1-0.01)} = \frac{25}{99}$$

Unit 5 Day 4

Test for Divergence

Telescopic Test

P-Series (Including Harmonic)

QUIZ Tomorrow

- Quiz Review—packet p. 5 & 6
  - We will have some POD time tomorrow before the quiz but you might want to start on this tonight!
- The quiz covers:

  Reducing factorials,

  Convergence/divergence of:

  SEQUENCE

  Geometric Series

  Divergence Test—TODAY

  p-Series Test—TODAY

Test for **Divergence** Theorem

If $\lim_{n\to\infty} a_n$ Does Not Exist

OR if $\lim_{n\to\infty} a_n = \pm\infty$

OR if $\lim_{n\to\infty} a_n \neq 0$

Then the series $\sum_{n=1}^{\infty} a_n$ is divergent

Test for Divergence

$\sum_{n=1}^{\infty} \frac{n^2+2n}{3-n^2}$	$\lim_{n\to\infty} \frac{n^2+2n}{3-n^2} = -1$	*Series Diverges*
$\sum_{n=1}^{\infty} \frac{n^2+2n}{n}$	$\lim_{n\to\infty} \frac{n^2+2n}{n} = \infty$	*Series Diverges*
$\sum_{n=1}^{\infty} \frac{n}{n^2+2n}$	$\lim_{n\to\infty} \frac{n}{n^2+2n} = 0$	*Inconclusive*
$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$	$\lim_{n\to\infty} \frac{2}{n^2+4n+3} = 0$	*Inconclusive*

Practice

- Now you are ready to try

Packet p. 3, #3b, #14, 15, 17, 18, 22, 23, 24

Summary So Far

- Geometric Series
  - Examine *r* value
- Test for Divergence
  - Divergent OR INCONCLUSIVE
- Next . . . . P-Series (Including Harmonic Series)

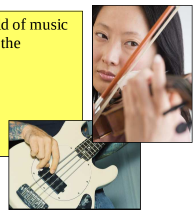
Exploring the Harmonic Series

$$\text{Harmonic Series} = \sum_{n=1}^{\infty}\frac{1}{n} = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...$$

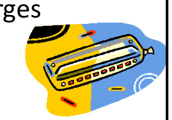
The name "Harmonic series" comes from the world of music and overtones, or harmonics. The wavelengths of the overtones of a vibrating string are

$\frac{1}{2},\frac{1}{3},\frac{1}{4}...$

Source: Wikipedia.com



Determine if the harmonic series converges or diverges.



$$\sum_{n=1}^{\infty}\frac{1}{n} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...$$

First, let's group

$$1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{16}\right)+...$$

$$1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{16}\right)+...$$

Is Greater Than

$$1+\frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+\left(\frac{1}{16}+\frac{1}{16}+...+\frac{1}{16}\right)+...$$

Which equals: $1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+...$

Which continues to grow

So, $\sum_{n=1}^{\infty}\frac{1}{n}$ must diverge.

The Harmonic Series is an example that confirms the converse of the following theorem to be false!

If the series

$$\sum_{n=1}^{\infty} a_n$$

is convergent,

then

$$\lim_{n\to\infty} a_n = 0$$

The CONVERSE is NOT TRUE

Retry p. 3 #15 and #27

$\#15. \sum_{n=1}^{\infty} \frac{3}{n}$

$\#27. \sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n} \right)$

The Harmonic Series is an example of a P-Series

A P-Series is of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$

P = 1 gives the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent IF $p > 1$

and divergent IF $p \le 1$

Determine convergence or divergence

Diverge			Converge	
$\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$	$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$	$\sum_{n=1}^{\infty} \frac{1}{n}$	$\sum_{n=1}^{\infty} \frac{1}{n^4}$	$\sum_{n=1}^{\infty} \frac{1}{n^3}$

$1. \sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$

$2. 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \ldots$

Summary So Far

- Geometric Series
  - Examine *r* value
- Test for Divergence
  - Divergent OR INCONCLUSIVE
- P-Series (Including Harmonic Series)
- NEXT . . . .Telescoping Test (NOT on Quiz #1)

Remember . . . .

If the sequence of the partial sums converges to some value S ($\lim_{n\to\infty} s_n = S$),

then the series $\sum a_n$ converges…

AND we state that

$$\sum_{n=1}^{\infty} a_n = S$$

Telescoping Test…Ex. P. 3 #19

$$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \sum_{n=1}^{\infty} \frac{-1}{n+3} + \sum_{n=1}^{\infty} \frac{1}{n+1}$$

$$\sum_{n=1}^{\infty} \frac{-1}{n+3} = \quad -\frac{1}{4} - \frac{1}{5} - \frac{1}{6} \ldots. - \frac{1}{n+3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n+1} = \quad \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots. + \frac{1}{n+1}$$

Let's look at partial sums:

Telescoping Test…Ex. P. 3 #19

The *n*th partial sum is

$$S_n = \frac{1}{2} + \frac{1}{3} + \left(-\frac{1}{n+2}\right) + \left(-\frac{1}{n+3}\right)$$

And

$$\lim_{n\to\infty} S_n = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

So the series

$$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$$

Converges to 5/6

Example #2—NOT in your packet
YOU TRY

$$\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$$

$$\sum_{n=1}^{\infty} \frac{2}{4n^2-1} = \sum_{n=1}^{\infty} \frac{1}{2n-1} - \sum_{n=1}^{\infty} \frac{1}{2n+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots + \frac{1}{2n-1}$$

$$-\sum_{n=1}^{\infty} \frac{1}{2n+1} = -\frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \ldots + \left(-\frac{1}{2n+1}\right)$$

Example #2—NOT in your packet SOLUTION

The *n*th partial sum is

$$S_n = 1 - \frac{1}{2n+1}$$

And

$$\lim_{n\to\infty} S_n = 1 - \frac{1}{2n+1} = 1$$

So the series

$$\sum_{n=1}^{\infty} \frac{2}{4n^2 - 1}$$

Converges to 1

Retry Packet p. 3 #26

$$\sum_{n=1}^{\infty} \left(\frac{3}{n(n+3)} + \frac{5}{4^n} \right)$$

Later Go Back and Try . . . .

- Packet p. 3 #16