Squeeze Theorem for Sequences

$$\text{If } a_n \le b_n \le c_n \text{ for } n \ge n_0$$

$$\text{and } \lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$$

$$\text{then } \lim_{n\to\infty} b_n = L$$

EXAMPLE: (Packet p. 2 #31)
Determine whether the sequence converges or diverges. If it converges, find the limit.

$$31.\ a_n = \frac{\cos^2 n}{2^n}$$

HW Questions

Important Clarification. . .
What does diverge mean?

Diverge does not automatically mean sequence goes to an infinity.

A diverging sequence could go to an infinity or it could be oscillating, such as with a_n = *sin(n)*.

Alternating Sequences-Signs of Terms Alternate + and -

$$(-1)^n \quad or \quad (-1)^{n+1}$$

Alternating sequences don't have to be divergent. The sequence could be alternating but still converging to the same number.

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \cdots \qquad \text{converges to } 0$$

Objectives

- **Terminology**
  - Sequence, series, partial sum
  - Summation Notation
- **Geometric Series**
  - What it is
  - Convergence test
  - Sum calculation

Sequence *versus* Series

Sequence:
A list of terms

$$\{a_1, a_2, \ldots a_n\}$$

Series:
A SUM of terms

$$a_1 + a_2 + a_3 + \ldots\ldots + a_n$$

PARTIAL SUMS:

$$s_1 = a_1$$
$$s_2 = a_1 + a_2$$
$$s_3 = a_1 + a_2 + a_3$$

In general:

$$s_n = a_1 + a_2 + a_3 + \ldots + a_n$$

Summation Notation:

$$\sum_{k=1}^{n} a_k = s_n$$

This stands for the sum of a_k from k = 1 to k = n

<u>Examples:</u>
Find the first 3 terms for the following series. Also, find s_3

$$\sum_{k=1}^{n} 3k$$

Find the sixth term for the following series.

$$\sum_{k=3}^{n} (1+k^2)$$

A series can converge or diverge

If the sequence of the partial sums converges to some value S ($\lim_{n\to\infty} s_n = S$)
then the series $\sum a_n$ converges
AND we state that $\sum_{n=1}^{\infty} a_n = S$

Infinite Geometric Series

Each term differs by the same common ratio.
And there are an infinite number of terms

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \ldots + ar^{k-1} + \ldots$$

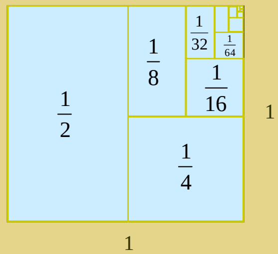
If r =1 what does the series look like?

$$a + a + a + a + \ldots = \infty a$$

The series diverges because it never approaches a limiting value.
We have infinite number of "a's".

Now, let's explore a converging infinite geometric series.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\cdots=1$$



$$\sum_{k=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{k-1}=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+....$$

This series converges--approaches a limiting value of 1.

→

Finding out what any convergent geometric series will approach . . . . JUST Watch for now—I will tell you when to write.

$$s_n = a + ar + ar^2 + ar^3 + ... + ar^{n-1}$$

Mult. by r

$$rs_n = \quad ar + ar^2 + ar^3 + ... + ar^{n-1} + ar^n$$

Now, subtract the 2 eq.

$$s_n - rs_n = a - ar^n$$

$$s_n(1-r) = a - ar^n$$

$$s_n = \frac{a - ar^n}{(1-r)}$$

$$s_n = \frac{a - ar^n}{(1-r)}$$

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}\frac{a-ar^n}{(1-r)} = \lim_{n\to\infty}\frac{a(1-r^n)}{(1-r)}$$

Since –1 < r < 1, then as n increases w/out bound . . . $\lim_{n\to\infty} r^n = 0$

$$s_n = \frac{a}{(1-r)}$$

Now add to your notes . . .

An INFINITE geometric series such as

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ...$$

Is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

If $|r| \geq 1$, the geometric series is divergent

Ex: Is the following series convergent or divergent? If convergent, find its sum.

$$\sum_{n=1}^{\infty} 6\left(\frac{2}{3}\right)^n$$

More Practice

- **Packet p. 3 #10, 12, 20, 21**

#10. $$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$$