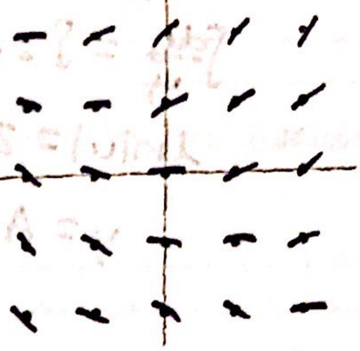
Day 3

Free Response Questions: Slope Fields and Euler's Method

1. Consider the differential equation $\frac{dy}{dx} = \frac{(x+y)}{4}$

   A) On the axes provided, sketch the slope field of the given differential equation.

n	x_n	y_n	$\frac{x+y}{4}$	$\frac{dy}{dx} \cdot 0.1$
0	0	4	1	0.1
1	0.1	4.1	1.05	0.105
2	0.2	4.205		



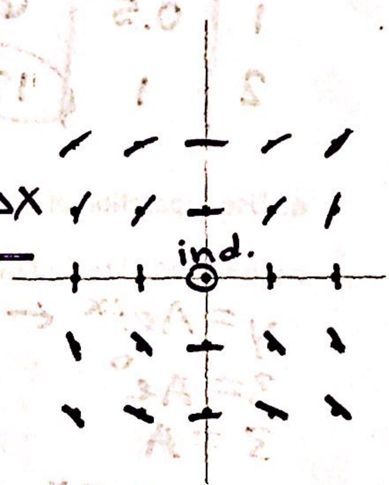
WORK

   B) Let $y = f(x)$ be the particular solution to the differential equation subject to the initial condition $f(0) = 4$. Use Euler's Method, starting at x=0, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to the answer.

$f(0.2) \approx 4.205$

2. Consider the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$.

n	x_n	y_n	dy/dx	$dy/dx \cdot \Delta x$
0	0	-1	0	0
1	.1	-1	-0.01	-0.001
2	.2	-1.001		



WORK

   A) On the axes provided, sketch the slope field of the given differential equation.

   B) Let $y = f(x)$ be the particular solution to the differential equation subject to the initial condition $f(0) = -1$. Use Euler's method, starting at x=0, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to the answer. $f(.2) \approx -1.001$

   C) Find the particular solution to the differential equation with the initial condition $f(0) = -1$.

$\int y\,dy = \int x^2\,dx$

$y^2/2 = x^3/3 + C$

$1/2 = 0 + C \rightarrow C = 1/2$

$\frac{y^2}{2} = \frac{x^3}{3} + \frac{1}{2}$

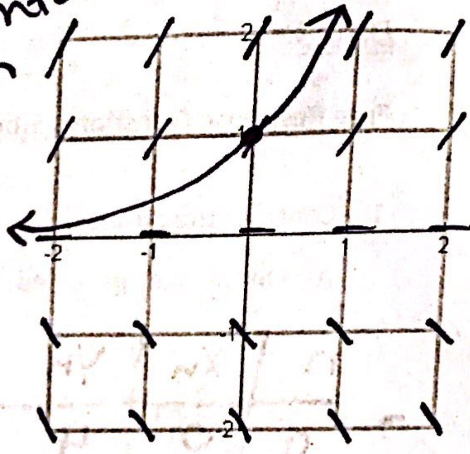
$3y^2 = 2x^3 + 3$

$y^2 = \frac{2x^3+3}{3}$

$\boxed{y = \sqrt{\frac{2x^3+3}{3}}}$

3. Given the differential equation $\frac{dy}{dx} = 2y$

Horizontal Pattern



a. Make the slope field on the grid to the right.

b. Draw a possible curve through (0,1).

$\int \frac{dy}{y} = \int 2\,dx$ $\quad 1 = Ae^0$

$\ln|y| = 2x + C$ $\quad A = 1$

$y = Ae^{2x}$ $\quad \boxed{y = e^{2x}}$

c. Use Euler's method with an initial condition of (0,3) and a $\Delta x = 0.5$ to estimate the y value at x = 1.

0.5

n	x_n	y_n	$dy/dx = 2y$	$dy/dx \cdot \Delta x$
0	0	3	6	3
1	0.5	6	12	6
2	1	12		

d. Use separation of variables to solve the differential equation $\frac{dy}{\partial x} = 2y$ with initial condition (0,3) to find the exact value at x = 1.

$y = Ae^{2x}$ ← (see #3b, first part)

$3 = Ae^0$

$3 = A$ $\qquad y(1) = 3e^2 \approx \boxed{22.167}$

$y = 3e^{2x}$

e. Is your answer from part (c) an over or underestimate? Explain.

Approx. from Euler's Method $= \boxed{12 < 3e^2} =$ Actual