AP Calculus BC

Unit 1 Day 2

Arrival Activity

- With a partner, discuss any issues you had with last night's homework

- AND answer the following using the same graph that was given for HW #1-8
  - What is the point that is located at $-\pi/6$?

- QUIZ today!!!!

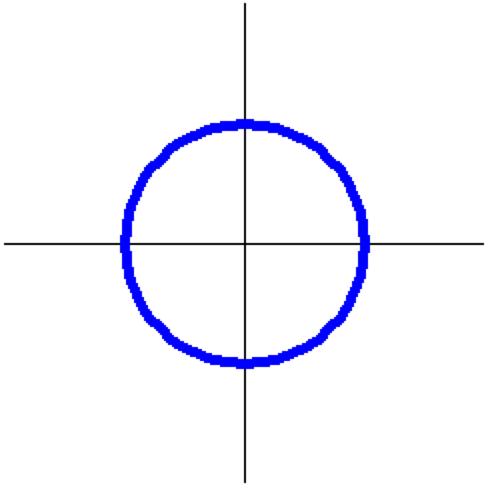
HW Questions

Next . . . . .

- Continuation of yesterday’s notes/topic

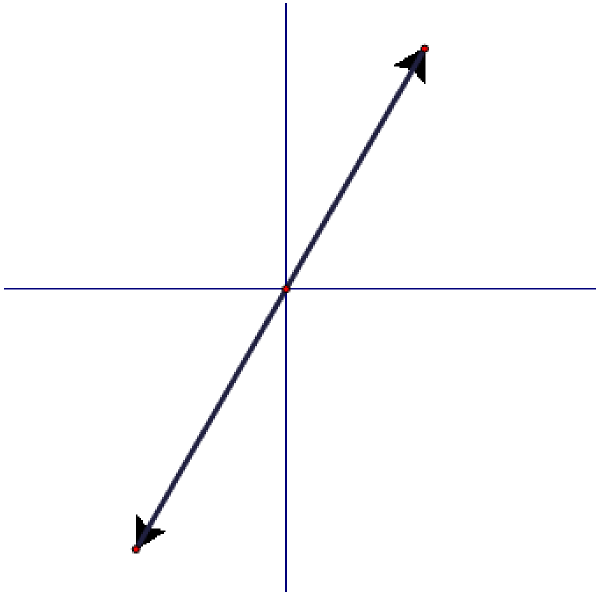
Referring to yesterday’s notes, what do the graphs of the following look like.

$r = -1$



Can you think of another equation that would result in the same graph?

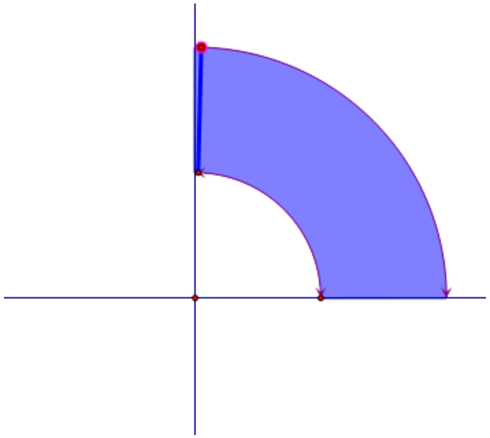
$\theta = \frac{\pi}{3}$



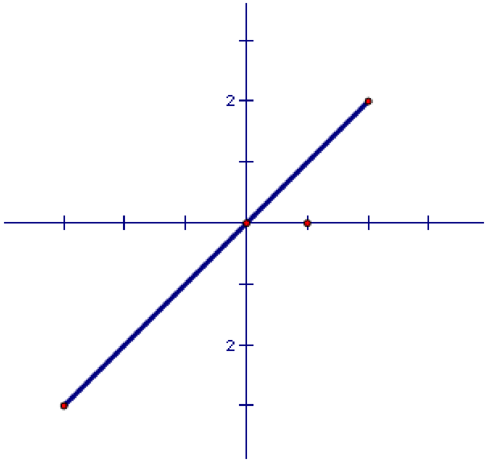
Can you think of two other equations that would result in the same graph?

What would the graph of these **pairs** of equations look like?

A. $1 \le r \le 2 \;\; and \;\; 0 \le \theta \le \dfrac{\pi}{2}$

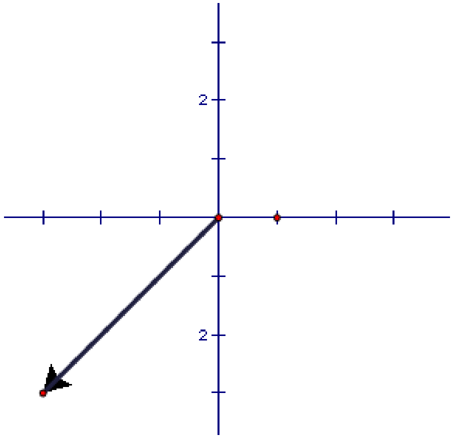


B. $-3 \le r \le 2 \;\; and \; \theta = \dfrac{\pi}{4}$



How about this pair ?

$$C. \quad r \leq 0 \;\; and \; \theta = \frac{\pi}{4}$$



AP Calculus BC

NEW MATERIAL

Polar vs Cartesian

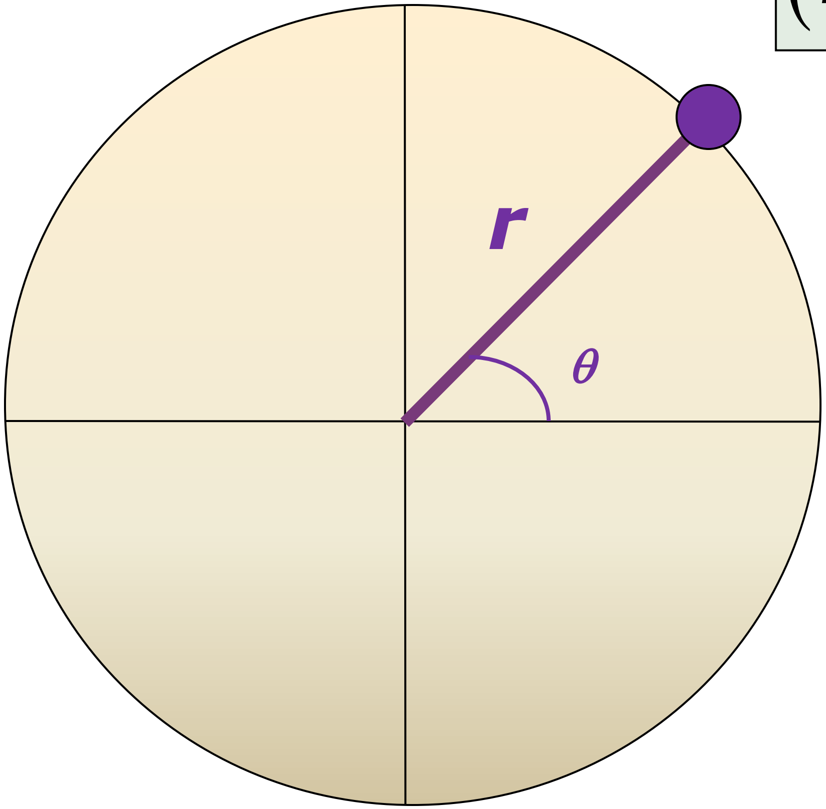
Unit 1 Day 2

It is good to have a Cartesian coordinate system AND a Polar coordinate system because some functions are easier in Cartesian and some are easier in Polar:

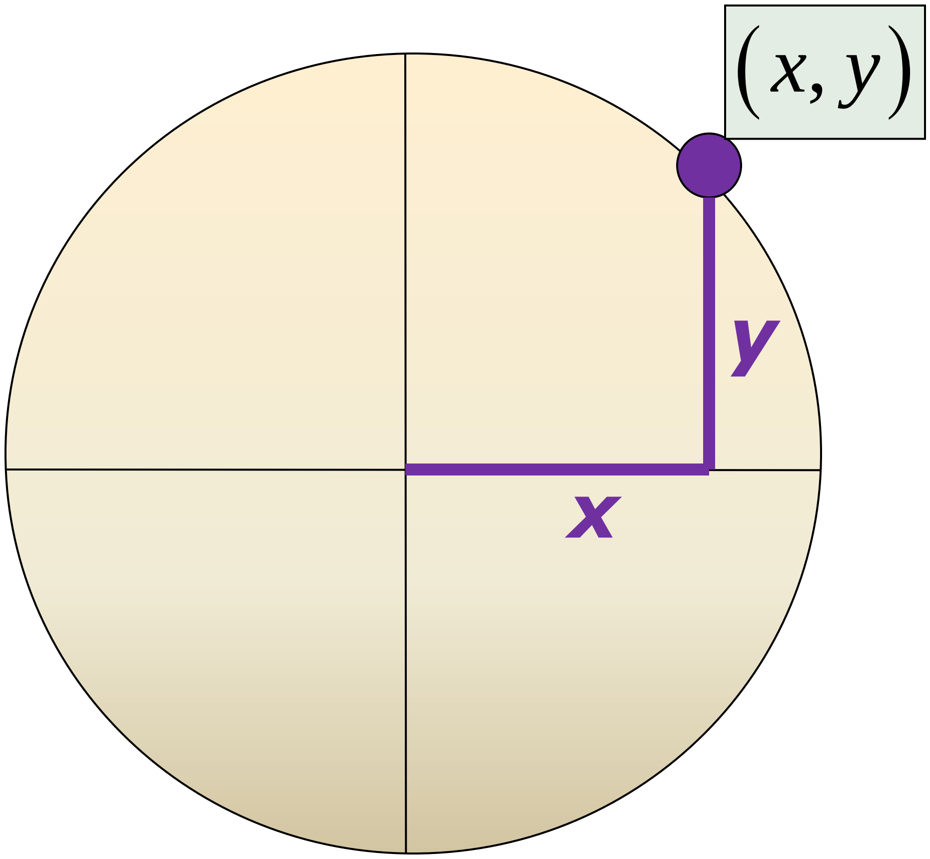
Polar	Cartesian
$r\cos\theta = 2$	$x = 2$
$r^2\cos\theta\sin\theta = 4$	$xy = 4$
$r^2\cos^2\theta - r^2\sin^2\theta = 1$	$x^2 - y^2 = 1$
$r = 1 + 2r\cos\theta$	$y^2 - 3x^2 - 4x - 1 = 0$
$r = 1 - \cos\theta$	$x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - y^2 = 0$

Recall from yesterday . . . .

$$(r, \theta)$$

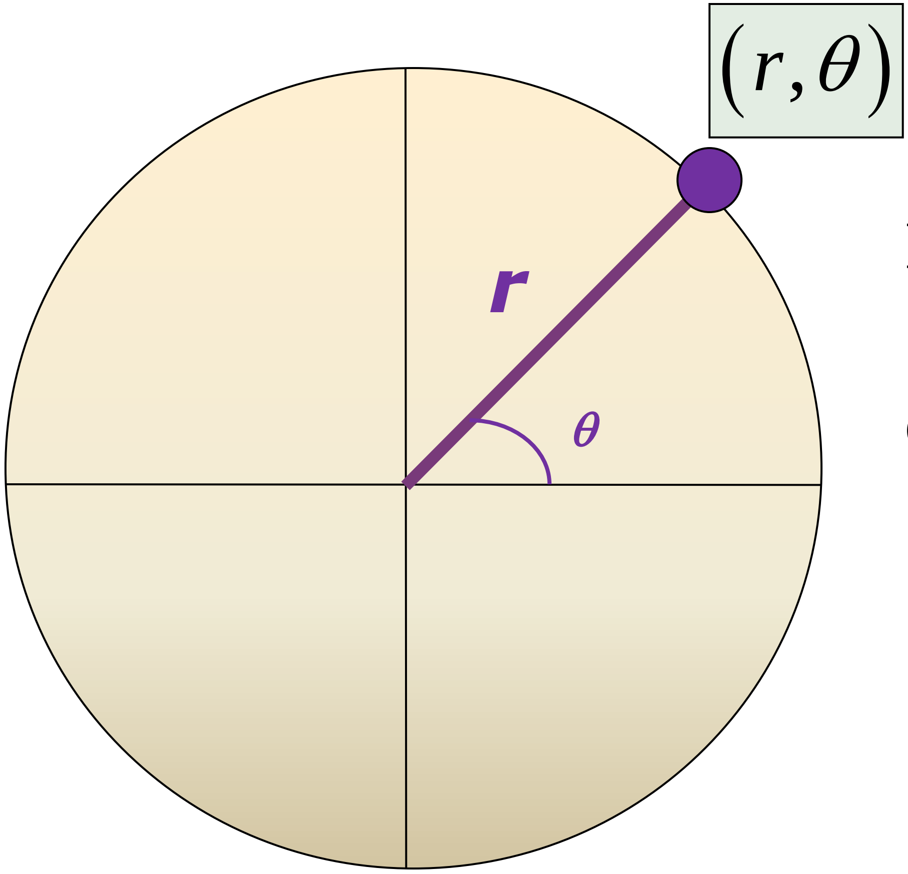


Same point but different coordinate system . . .



Converting from polar to Cartesian:

$$x = r\cos\theta$$
$$y = r\sin\theta$$



Example:

Convert $\left(4, \frac{\pi}{6}\right)$ **to Cartesian coordinates** (x, y) **:**

Now, YOU try!!

Convert into Cartesian:

1) $\left(4, \frac{7\pi}{3}\right)$

2) $\left(-3, \frac{5\pi}{4}\right)$

3) $\left(-5, \frac{\pi}{6}\right)$

$Answers:$

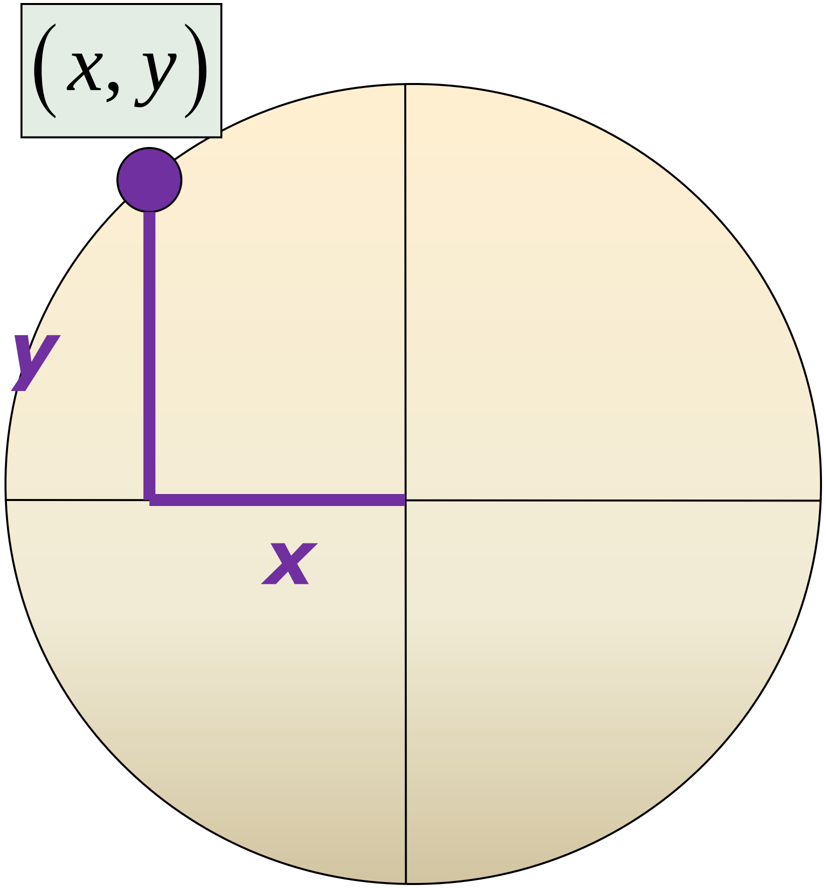
1) $\left(2, 2\sqrt{3}\right)$

2) $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

3) $\left(\frac{-5\sqrt{3}}{2}, -\frac{5}{2}\right)$

Converting from Cartesian to Polar

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$



Example:

Convert $\left(-3, \sqrt{3}\right)$ **to Polar Coordinates**

Finding r:

Finding θ is more involved . . .

Finding θ . . . .

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

Example:

Convert $\left(-3, \sqrt{3}\right)$ **to Polar Coordinates**

Now we have to pair up the r and θ . . .

Pairing up r and θ . . .

$$r = \pm\sqrt{12} \qquad \theta = \frac{11\pi}{6} \text{ or } \theta = \frac{5\pi}{6}$$

The original Cartesian coordinates place the point in the 2^{nd} quadrant.

So . . . .

The Cartesian point $\left(-3, \sqrt{3}\right)$ in Polar form is:

$$\left(2\sqrt{3}, \frac{5\pi}{6}\right) \quad \textbf{OR} \quad \left(-2\sqrt{3}, \frac{11\pi}{6}\right)$$

There are multiple conversions to polar! ☺

Being in the correct location when finished with the conversion is what is important.

Up Next . . .

Converting points that would not have a θ landing on a known locations on the unit circle

Convert $(-3,-4)$ to Polar Coordinates

$$r = \pm\sqrt{(-3)^2 + (-4)^2} = \pm 5$$

$$\theta = \tan^{-1}\left(\frac{-4}{-3}\right) = 0.927$$ This is in the first quadrant.

To end up in the 3rd quadrant, the location of the given point, we would need $(-5, 0.927)$

The other possibility would be to add π to the angle and use the positive value of *r*. $(5, 0.927 + \pi)$

Practice:
Convert into Polar Coordinates

$$1)\ \left(1, \sqrt{3}\right)$$

$$2)\ \left(1, -1\right)$$

$$3)\ \left(\sqrt{2}, -\sqrt{2}\right)$$

$$Answers:$$

$$(at\ least\ one\ of\ them)$$

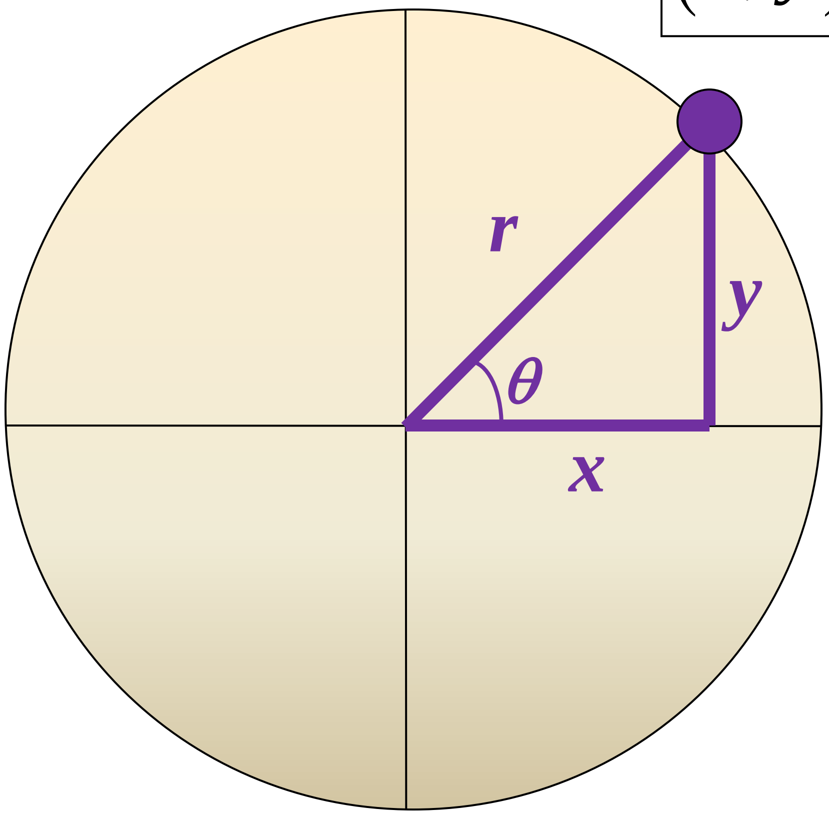
$$1)\ \left(2, \frac{\pi}{3}\right)$$

$$2)\ \left(\sqrt{2}, -\frac{\pi}{4}\right)$$

$$3)\ \left(2, \frac{7\pi}{4}\right)$$

Recap

$$(r, \theta)$$
$$(x, y)$$



Cartesian to Polar

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

Polar to Cartesian

$$\cos\theta = \frac{x}{r} \quad \Leftrightarrow \quad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \quad \Leftrightarrow \quad y = r\sin\theta$$

Converting EQUATIONS from Polar to Cartesian

$$1.\ r = -3\sec\theta$$

Confirm your answer by graphing the original polar equation to see that it is a vertical line at x=-3

2. $$r = \frac{5}{\sin\theta - 2\cos\theta}$$

Confirm your answer by graphing the original polar equation to see that it is a line equivalent to y=5+2x

QUIZ--Unit Circle BC Style

- AFTER quiz start on HW
  - Textbook pg. 738
    (1, 3, 5, 11, 13, 15, 27, 31, 33, 37, 41, and 45)