Unit 2 Parametrics
Day 2—Vectors in a Plane

WARMUP

1. Find the equation of the line tangent to the curve at the given value of *t.*

$$x = 5\cos t \quad y = 3\sin t \quad at \;\; t = \frac{\pi}{4}$$

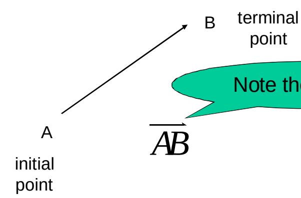
2. Determine $\frac{d^2y}{dx^2}$ of the following curve when *t=1.*

$$x = t^2 - 3t \quad y = t^3$$

3. Find the length of the curve from problem #2 when -1 ≤ *t* ≤ *2.*

HW Questions?

Quantities such as force, displacement or velocity have direction and magnitude. These quantities can be represented by directed line segments.



Note the symbol

$\overline{AB}$

The length, also known as the magnitude, is denoted as $\left|\overline{AB}\right|$

More later!

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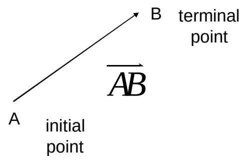

A vector in a plane is represented by a directed line segment. Textbooks use lowercase, **boldface** letters.

Equal vectors have the same length and direction (same magnitude and slope).

More later!

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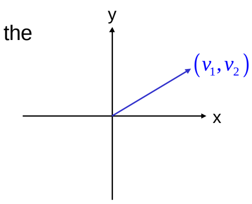
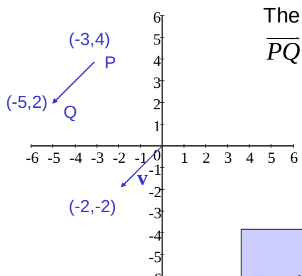
A vector is in standard position if the initial point is at the origin.



The component form of this vector is: $\mathbf{v} = \langle v_1, v_2 \rangle$

The magnitude (length) of $\mathbf{v} = \langle v_1, v_2 \rangle$ is:

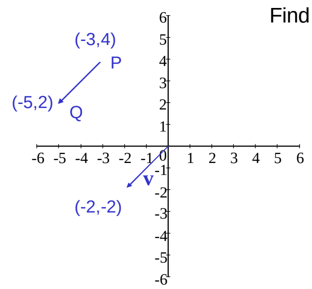
$$|\mathbf{v}| = \sqrt{{v_1}^2 + {v_2}^2}$$



The component form of $\overline{PQ}$ is:

Order matters!

Terminal Point – Initial Point



Find the magnitude of $\overline{PQ}$

OTHER important terms

If $|\mathbf{v}|=1$ then **v** is a unit vector.

$\langle 0,0\rangle$ is the zero vector and has no direction.

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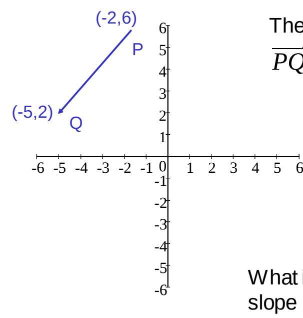
Slopes and Vectors



The component form of $\overline{PQ}$ is: $\mathbf{v}=\langle -3,-4\rangle$

What is the slope of PQ?

$$m=\frac{2-6}{-5-(-2)}=\frac{-4}{-3}$$

What is the connection between the slope and component form?

$$\frac{dy}{dx}=\langle dx,dy\rangle$$

Think about it.

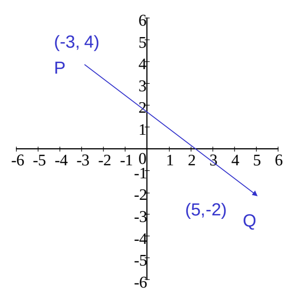
$$\text{Slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{dy}{dx}$$

$$\text{Vector}=\langle x_2-x_1,y_2-y_1\rangle=\langle dx,dy\rangle$$

To ensure direction and magnitude are preserved:

- Do not reduce or cancel signs in slope calculation
- It's terminal – initial in slope calculation.

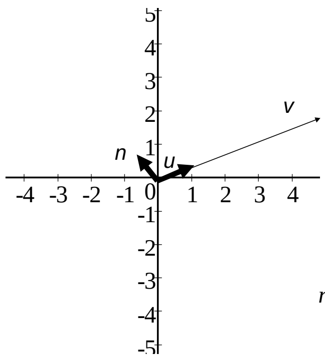
Find the slope and component form of the vector.



$$\frac{dy}{dx}=\frac{-6}{8}$$

$$\overline{PQ}=\langle 8,-6\rangle$$

Parallel and Normal Unit Vectors



Vector *u* is a unit vector parallel to vector *v*.

$$u = \frac{v}{|v|} = \left\langle \frac{v_1}{|v|}, \frac{v_2}{|v|} \right\rangle$$

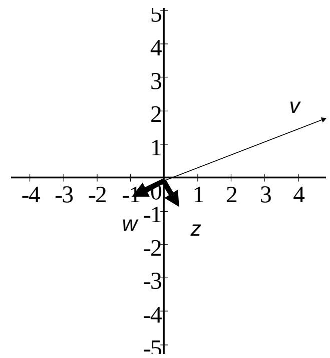
Vector *n* is a unit vector normal to vector *v*.

$$n = \langle -u_2, u_1 \rangle$$

Opposite

Reciprocal

Parallel and Normal Unit Vectors



w and z are also tangent and normal vectors to *v.*

Example Problem

Find the unit vectors that are tangent and normal to the following parametrized curve at the point where t=4.

$$x = \frac{t}{2} + 1, \quad y = \sqrt{t} + 1, \quad t \geq 0$$

Vector Operations:

Let $\mathbf{u} = \langle u_1, u_2 \rangle$, $\mathbf{v} = \langle v_1, v_2 \rangle$, k is a scalar (real number).

$$\mathbf{u} + \mathbf{v} = \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$$

(Add the components.)

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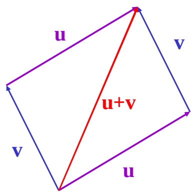

u + **v** is the resultant vector.

(Parallelogram law of addition)

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Vector Operations:

$$\mathbf{u}-\mathbf{v}=\langle u_1,u_2\rangle-\langle v_1,v_2\rangle=\langle u_1-v_1,u_2-v_2\rangle$$

(Subtract the components.)

Scalar Multiplication: $k\mathbf{u}=\langle ku_1,ku_2\rangle$

Negative (opposite): $-\mathbf{u}=(-1)\mathbf{u}=\langle -u_1,-u_2\rangle$

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Example Problem

Let $\mathbf{u}=\langle -1,3\rangle$ and $\mathbf{v}=\langle 4,7\rangle$.

Find the (a) component form and (b) magnitude of the the following:

$$2\mathbf{u}+3\mathbf{v}$$