WARMUP:

A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train that is traveling at 60 meters per second. At what rate is the distance from the observer to the train changing when 4 seconds have passed since the train went through the intersection?

A)57.60 B) 57.88 C) 59.20 D) 60.00 E) 67.40

If *y=2x-8*, what is the minimum value of the product of xy?

A) -16 B) -8 C) -4 D) 0 E) 2

WARMUP:

A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train that is traveling at 60 meters per second. At what rate is the distance from the observer to the train changing when 4 seconds have passed since the train went through the intersection?

A)57.60 B) 57.88 C) 59.20 D) 60.00 E) 67.40

If *y=2x-8*, what is the minimum value of the product of xy?

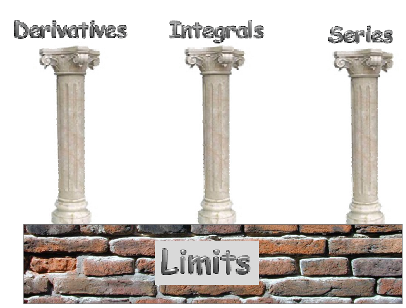
A) -16 B) -8 C) -4 D) 0 E) 2

BC Calculus

Today's Topic

Sequences—the building blocks of Series

3 Pillars of Calculus AB/BC



BC Calculus

Today:
Review of Precalculus Topic of Sequences with some new notation

What's the next term?

3, 6, 12, 24, …

This is a <u>sequence</u>—A list of numbers that follow a pattern.

Definition of a SEQUENCE
(with some mathematical notation):

A sequence is a list or set of terms

$$\{a_1, a_2, ..., a_n\}$$

where $n \in Z^+$

"is an element of" "the set of positive integers"

{3, 6, 12, 24, …}

Value of 1st term - $a_1 = 3$

Value of 2nd term - $a_2 = 6$

Value of nth term - a_n

Value of "next term": a_{n+1}

Value of "previous term": a_{n-1}

Sequences are generated by formulas:

Recursive – The value of an element is a function of the **value of the previous element**

$$a_n = f\left(a_{n-1}\right) \;\; or \;\; a_{n+1} = f\left(a_n\right)$$

Explicit – The value of an element is a function of the **position of the element** in the sequence.

$$a_n = f\left(n\right)$$

Packet p. 2 #11-14

Find a formula for the general term a_n of the following sequence, assuming the pattern of the first few terms continues

11. $\{2,\ 7,\ 12,\ 17, \ldots.\}$

Use $a_n = a_1 + (n-1)d$, where d is the common difference between terms

Packet p. 2 #11-14

Find a formula for the general term a_n of the following sequence, assuming the pattern of the first few terms continues

12. $\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \ldots.\right\}$

Brainstorm with your partner.
1. What might we use to get the alternating signs?
2. Think about the position of the term when determining the value of the term.

Packet p. 2 #11-14

Find a formula for the general term a_n of the following sequence, assuming the pattern of the first few terms continues

13. $\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots.\right\}$

Use $a_n = a_1 (r)^{n-1}$,where r is the common ratio between terms

Packet p. 2 #11-14

Find a formula for the general term a_n of the following sequence, assuming the pattern of the first few terms continues

$$14. \quad \{5, 1, 5, 1, 5, 1, ...\}$$

Hint: Average two consecutive terms together then think about the relationship each term has to this average value.

Recursive Example: Find the first three elements of the sequence

$$a_1 = 1, \; a_{n+1} = \frac{a_n + 2}{a_n}$$

Explicit Examples: Find the first three elements of each sequence

$$\left\{\frac{n}{n^2+1}\right\} =$$

$$\left\{\frac{2^n}{n!}\right\} =$$

$$\left\{1+(-1)^n\right\} =$$

More Terminology:

A sequence is ***increasing*** if

$$a_{n+1} > a_n \quad \forall n \geq 1$$

"for all"

A sequence is ***decreasing*** if

$$a_{n+1} < a_n \quad \forall n \geq 1$$

A *monotonic* sequence is either increasing or decreasing.

Terminology Continued:

A sequence is ***bounded*** above if:

$$a_n < U \quad \forall n$$

No term is larger than U

Or below if:

$$a_n > L \quad \forall n$$

No term is smaller than L

Still More Terminology

A sequence has a ***limit*** if we can get arbitrarily close to a number L

$$\lim_{n\to\infty} a_n = L \qquad or \qquad a_n \to L \;\; as \;\; n \to \infty$$

If the limit exists, we say the sequence CONVERGES. Otherwise it DIVERGES.

Examples:

$\{2,4,8,16,\ldots\}$ Diverges because each element is larger than the one before it.

$\left\{1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\ldots\right\}$ Converges to the number 0.

$a_n = \sin(n)$ Diverges. Even though sin(n) is bounded, it oscillates and never settles on any single number.

Relationship between Sequence and Function:

$$If \;\; f(n) = a_n \;\; when \;\; n \;\; is \;\; an \; \text{integer}$$
$$and \lim_{x\to\infty} f(x) = L \;\; ,$$
$$then \lim_{n\to\infty} a_n = L.$$

In other words,

If there is a function that defines the sequence and that function has a limit, then the limit of the function is also the limit of the sequence.

And some more…boy these sound familiar…

$$\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$
$$\lim_{n\to\infty}(a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$$
$$\lim_{n\to\infty} ca_n = c\lim_{n\to\infty} a_n$$
$$\lim_{n\to\infty}(a_n b_n) = \lim_{n\to\infty} a_n \bullet \lim_{n\to\infty} b_n$$
$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad if \ \lim_{n\to\infty} b_n \neq 0$$
$$\lim_{n\to\infty} c = c$$

Practice: Determine if the sequence converges, or diverges. If it converges find the limit.

1. $\left\{\frac{1}{n}\right\}$
2. $\left\{1 + \frac{(-1)^n}{n}\right\}$
3. $\left\{\frac{2n^2}{3n^3 - 1}\right\}$
4. $\left\{\frac{3n^4 + 5}{4n^4 - 7n^2 + 9}\right\}$
5. $\left\{\frac{n^2 - 1}{n}\right\}$
6. $\{\sin n\}$
7. $\{(-1)^{n+1}\}$
8. $\left\{\frac{\ln n}{n}\right\}$

Answers:

1. 0 2. 1 3. 0 4. $\frac{3}{4}$

5. *diverges* (*to* ∞)

6. *diverges* (*but not to* ∞)

7. *diverges* (*oscillates*)

8. 0 (*using L'Hopital*)

Squeeze Theorem for Sequences

$$If \ a_n \le b_n \le c_n \quad for \ n \ge n_0$$
$$and \ \lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$$
$$then \ \lim_{n\to\infty} b_n = L$$

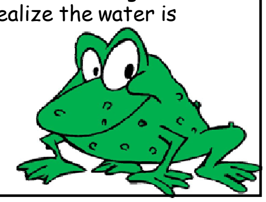
EXAMPLE: Packet p. 2 #31 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$31.\ a_n = \frac{\cos^2 n}{2^n}$$

A Story

A frog jumps into a pot of boiling water.
It jumps out because its hot.

Another frog jumps in a pot of cool water.
The water temperature is slowly raised to boiling.
The frog boils because it doesn't realize the water is getting too hot.



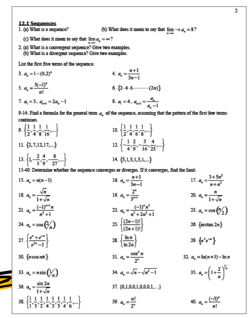
A Modern Telling

A Calculus student thinks what we cover each day is easy and decides not to do the homework.

On test day, the student looks at the first question and realizes he is now hopelessly lost and overwhelmed.

Moral: You need to keep up with the homework! If you don't, you will become overwhelmed without even realizing it. Don't become the boiled frog!

Packet p.2
"Sequences"
(CW-even) (HW-odds and finish evens)



2

12.1 Sequences

1. (a) What is a sequence? (b) What does it mean to say that $\lim_{n\to\infty} \to a_n = 8$?
   (c) What does it mean to say that $\lim_{n\to\infty} a_n = \infty$?
2. (a) What is a convergent sequence? Give two examples.
   (b) What is a divergent sequence? Give two examples.

List the first five terms of the sequence.

3. $a_n = 1-(0.2)^n$
4. $a_n = \frac{n+1}{3n-1}$
5. $a_n = \frac{3(-1)^n}{n!}$
6. $\{2\cdot4\cdot6\cdots\cdots(2n)\}$
7. $a_1 = 3,\ a_{n+1} = 2a_n - 1$
8. $a_1 = 4,\ a_{n+1} = \frac{a_n}{a_n - 1}$

9-14: Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

9. $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\ldots\right\}$
10. $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\ldots\right\}$
11. $\{2,7,12,17,\ldots\}$
12. $\left\{-\frac{1}{4},\frac{2}{9},-\frac{3}{16},\frac{4}{25},\ldots\right\}$
13. $\left\{1,-\frac{2}{3},\frac{4}{9},-\frac{8}{27},\ldots\right\}$
14. $\{5,1,5,1,5,1,\ldots\}$

15-40: Determine whether the sequence converges or diverges. If it converges, find the limit.

15. $a_n = n(n-1)$
16. $a_n = \frac{n+1}{3n-1}$
17. $a_n = \frac{3+5n^2}{n+n^2}$
18. $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$
19. $a_n = \frac{2^n}{3^{n+1}}$
20. $a_n = \frac{n}{1+\sqrt{n}}$
21. $a_n = \frac{(-1)^{n-1}n}{n^2+1}$
22. $a_n = \frac{(-1)^n n^3}{n^3+2n^2+1}$
23. $a_n = \cos\left(\frac{n}{2}\right)$
24. $a_n = \cos\left(\frac{2}{n}\right)$
25. $\left\{\frac{(2n-1)!}{(2n+1)!}\right\}$
26. $\{\arctan 2n\}$
27. $\left\{\frac{e^n+e^{-n}}{e^{2n}-1}\right\}$
28. $\left\{\frac{\ln n}{\ln 2n}\right\}$
29. $\{n^2e^{-n}\}$
30. $\{n\cos n\pi\}$
31. $a_n = \frac{\cos^2 n}{2^n}$
32. $a_n = \ln(n+1) - \ln n$
33. $a_n = n\sin\left(\frac{1}{n}\right)$
34. $a_n = \sqrt{n} - \sqrt{n^2-1}$
35. $a_n = \left(1+\frac{2}{n}\right)^{1/n}$
36. $a_n = \frac{\sin 2n}{1+\sqrt{n}}$
37. $\{0,1,0,0,1,0,0,0,1,\ldots\}$
38. $\left\{\frac{1}{1},\frac{1}{3},\frac{1}{2},\frac{1}{4},\frac{1}{3},\frac{1}{5},\frac{1}{4},\frac{1}{6},\ldots\right\}$
39. $a_n = \frac{n!}{2^n}$
40. $a_n = \frac{(-3)^n}{n!}$

Together lets do #6 and #32

Hints:
Remember L'Hopital's Rule

And for #33 and 34 you will need to convert to a "familiar" indeterminate form.