UNIT 3 DAY 2
L'HOPITAL'S RULE

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1. The function f is continuous on the closed interval [1, 3] and has the values given in the table. The equation $f(x) = 5/4$ must have at least two solutions in the interval [1, 3] if $k =$

x	1	2	3
$f(x)$	2	k	4

A) 1/4
B) 3/2
C) 2
D) 9/4
E) 3

2. If $f(x) = \tan 3x$, then $f'(\frac{\pi}{9}) =$

A) 4/3
B) 4
C) 6
D) 12
E) $6\sqrt{3}$

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HW QUESTIONS??

Unit 3 Day 2
L'Hopital's Rule

We have spent a little time reviewing limits from Calculus AB…

1. $\lim_{x \to 2} \frac{x^2 + 6x - 16}{x - 2}$

2. $\lim_{x \to 0} \frac{\sin x}{x}$

3. $\lim_{x \to 3} \frac{\frac{1}{3} - \frac{1}{x}}{x - 3}$

Now, Limits in BC Calc

L'Hopital's Rule
& Indeterminate Forms

What happens near x=1 for the following function?

$$h(x) = \frac{\ln x}{x - 1}$$

Look at the Graph

Limit using Direct Substitution . . .

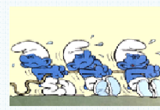
$$\lim_{x\to 1}\frac{\ln x}{x-1}=\frac{0}{0}$$

Direct substitution leads to : $\frac{0}{0}$

An INDETERMINATE form

Indeterminate Form $\frac{0}{0}$

- Maybe it's zero
- Maybe it's infinity or undefined
- Maybe it's some other number
- We don't know…yet


$\frac{0}{0}$


L'Hopital's Rule

If f(a)=g(a)=0 AND and f'(a) and g'(a) exist then . . . .

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=\frac{f'(a)}{g'(a)}$$

Why does it work? Here is a proof:

Start with the right side:

$$\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$$=\frac{f'(a)}{g'(a)}=\frac{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}{\lim_{x\to a}\frac{g(x)-g(a)}{x-a}}$$

$$=\lim_{x\to a}\frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}=\lim_{x\to a}\frac{f(x)-0}{g(x)-0}=\lim_{x\to a}\frac{f(x)}{g(x)}$$

Back to our problem . . . .

$$\lim_{x\to 1}\frac{\ln x}{x-1} = \lim_{x\to 1}\frac{\frac{1}{x}}{1} = \lim_{x\to 1}\frac{1}{x}$$

$$=1$$

Let's revisit the limit review problems . . .

Graphically we could tell that $\lim_{x\to 0}\frac{\sin x}{x}=1$

Today we can show this analytically using L'Hopital's Rule.

FIRST check to see that L'H can be applied!!

$$\lim_{x\to 0}\frac{\sin x}{x}=$$

You Try . . . .

$$\lim_{x\to 0}\frac{1-\cos x}{x}=$$

L'Hopital's Rule

Also applies to

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

Another indeterminate form

Remember when using L'Hopital's Rule:

Always check for an indeterminant form!

Calculate $\lim_{x\to\infty}\frac{e^x}{x^2}$

Indeterminant form? $\lim_{x\to\infty} e^x = \infty$ $\lim_{x\to\infty} x^2 = \infty$

$$\lim_{x\to\infty}\frac{e^x}{x^2} = \lim_{x\to\infty}\frac{e^x}{2x}$$

$\lim_{x\to\infty} e^x = \infty$

We still have an indeterminate form… $\lim_{x\to\infty} 2x = \infty$

So, let's do it again!

$$\lim_{x\to\infty}\frac{e^x}{x^2} = \lim_{x\to\infty}\frac{e^x}{2x} = \lim_{x\to\infty}\frac{e^x}{2} = \infty$$

Incognito Indeterminate Form:

$\frac{0}{0}$ *or* $\frac{\pm\infty}{\pm\infty}$ Are the familiar indeterminate forms.

An indeterminate form could be in disguise:

$\infty \bullet 0,\ \infty - \infty,\ 1^\infty, 0^0, or\ \infty^0$

The goal will be to manipulate the expression until it looks "familiar"

Ex. $\lim_{x\to\pi^-}(x-\pi)\cot x$ $= 0\cdot(-\infty)$

An incognito indeterminate form.

We need to transform it to be familiar.

$$\lim_{x\to\pi^-}(x-\pi)\cot x = \lim_{x\to\pi^-}\frac{x-\pi}{\tan x} = \frac{0}{0}$$

Now we can use L'H

$$\lim_{x\to\pi^-}\frac{x-\pi}{\tan x} = \lim_{x\to\pi^-}\frac{1}{\sec^2 x} = \lim_{x\to\pi^-}\cos^2 x = 1$$

Strategy we used: Rewrite multiplication as division

Next Example . . .

$$\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=\infty-\infty$$

Strategy: Rewrite with a common denominator.

THEN use L'H if possible.

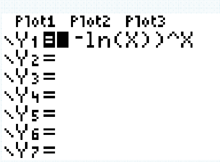
A little more challenging . . .

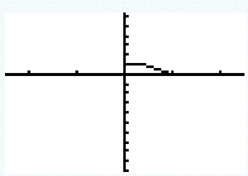
Ex. $\lim_{x\to 0+}(-\ln x)^x$ $=\infty^0$

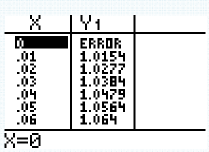
Indeterminate form but not "familiar"!!

So, can't use L'H Rule yet.

First let's look at the graph and the table of x and y values







Ex. $\lim_{x\to 0+}(-\ln x)^x$

Manipulate the expression we are taking the limit of . . .

$$let \quad f(x)=(-\ln x)^x$$

Then, taking the natural log of both sides . . .

$$\ln f(x)=x\ln(-\ln x)=\frac{\ln(-\ln x)}{\frac{1}{x}}$$

Looking at . . . $\lim_{x\to 0+} \ln f(x) = \lim_{x\to 0+} \frac{\ln(-\ln x)}{\frac{1}{x}} = \frac{\infty}{\infty}$

We can apply L-H Rule:

$$\lim_{x\to 0+} \ln f(x) = \lim_{x\to 0+} \frac{\ln(-\ln x)}{\frac{1}{x}} = \lim_{x\to 0+} \frac{\frac{1}{-\ln x}\left(-\frac{1}{x}\right)}{-x^{-2}}$$

$$= \lim_{x\to 0^+}\left(-\frac{x}{\ln x}\right)$$

$$= -\frac{0}{-\infty} = 0$$

Back to our original problem . . . .

$\lim_{x\to 0+} (-\ln x)^x =$

$\lim_{x\to 0+} f(x) =$ Substitute *f(x)*

$\lim_{x\to 0+} e^{\ln f(x)} =$ Properties of natural logarithms

$e^0 =$ Just found

1

Summary of Limit Forms

Determinate	**Indeterminate**
$\frac{0}{nonzero}$ or $\frac{0}{\pm\infty} = 0$	$\frac{0}{0}, \frac{\infty}{\infty}$
$\frac{\pm\infty}{nonzero} = \pm\infty$	$0 \cdot \infty$
$\frac{nonzero}{0} = \pm\infty$	$\infty - \infty$
$(nonzero)\cdot(\pm\infty) = \pm\infty$	$1^\infty, 0^0, \infty^0$

Summary of Techniques

$\frac{0}{0}, \frac{\infty}{\infty}$	Use L'Hopital directly
$0 \cdot \infty$	Rewrite as division, then use L'Hopital
$\infty - \infty$	Find a common denominator then use L'Hopital.
$1^\infty, 0^0, \infty^0$	Use *ln* to pull exponent out then use L'Hopital.