Day 2

In Exercises 5-8, use analytic methods to find the exact solution of the initial value problem. (*Hint:* This means separate and integrate.)

5. $y' = 1 + y,\ \ y(0) = 1$

$\int \frac{1}{1+y}\,dy = \int dx$

$\ln|1+y| = x + C$

$y = Ae^x - 1$

$\boxed{y = 2e^x - 1}$

6. $y' = x(1-y),\ \ y(-2) = 0$

$\int \frac{1}{1-y}\,dy = \int x\,dx$

$-\ln|1-y| = \frac{1}{2}x^2 + C$

$y = Ae^{-x^2/2} + 1$

$\boxed{y = -e^{-x^2/2+2} + 1}$

$0 = Ae^{-2} + 1$

$-1 = Ae^{-2}$

$A = -1 \cdot e^2$

7. $y' = 2y(x+1),\ \ y(-2) = 2$

$\int \frac{1}{y}\,dy = \int 2(x+1)\,dx$

$\ln|y| = x^2 + 2x + C$

$y = Ae^{x^2+2x}$

$\boxed{y = 2e^{x^2+2x}}$

8. $y' = y^2(1+2x),\ \ y(-1) = -1$

$\int y^{-2}\,dy = \int 1 + 2x\,dx$

$-\frac{1}{y} = x + x^2 + C$

$y = \frac{-1}{x + x^2 + C}$

$\boxed{y = \frac{-1}{x^2 + x + 1}}$

In Exercises 15 and 16,

(a) Find the solution to the initial value problem.
(b) Use your solution to part (a) to find the exact value indicated.
(c) Use Euler's method with a step size of 0.2 to approximate the requested value.
(d) Determine the error in the Euler approximation.

15. $y' = 2y^2(x-1),\ \ y(2) = -\frac{1}{2},\ \ x_0 = 2,$ Approximate $y(3)$

a) $\int y^{-2}\,dy = \int 2x - 2\,dx$

$\frac{-1}{y} = x^2 - 2x + C$

$y = \frac{-1}{x^2 - 2x + C}$

$\boxed{y = \frac{-1}{x^2 - 2x + 2}}$

b) $\boxed{y = -1/5} = -0.2$

$0.2 = 1/5$

c)

n	x_n	y_n	$2y^2(x-1)$	$y' \cdot (.2)$
0	2	$-1/2$	$2(1/4)(1) = 1/2$	$1/2(1/5) = 1/10$
1	2.2	$-2/5$	$2(4/25)(\frac{7}{5}) = 48/125$	$\frac{48}{625}$
2	2.4	$\frac{-202}{625}$	.2925	.0585
3	2.6	$-.265$	.2242	.0448
4	2.8	$-.2202$	.1745	.0349
5	3	$-.185$		

d) Error = $\boxed{.015}$

16. $y' = y - 1,\ \ y(0) = 3,\ \ x_0 = 0,$ Approximate $y(1)$

a) $\int \frac{1}{y-1}\,dy = \int dx$

$\ln|y-1| = x + C$

$y - 1 = Ae^x$

$y = Ae^x + 1$

$A = 2 \to \boxed{y = 2e^x + 1}$

b) $y = 2e + 1 \approx \boxed{6.437}$

c)

n	x_n	y_n	$y-1$	$y' \cdot (0.2)$
0	0	3	2	.4
1	.2	3.4	2.4	.48
2	.4	3.88	2.88	.576
3	.6	4.456	3.456	.6912
4	.8	5.1472	4.1472	.82944
5	1	$\boxed{5.97664}$		

d) Error $\approx \boxed{0.4599}$

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