Warmup

NO CALCULATORS

Convert into radians or degrees.

a) $45°$

b) $\frac{7\pi}{6}$

c) $-210°$

Find the exact value of:

a) $\cos(30°)$

b) $\sin\left(\frac{4\pi}{3}\right)$

c) $\cot(300°)$

Nice job. Now try these . . .

Solve for θ.

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\theta = \tan^{-1}\ 1$$

Problems in this column return ONE answer because asking for output of inverse trig FUNCTION

Solve for θ, where $0 \le \theta \le 2\pi$

$$\sin\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\tan\theta = 1$$

Problems in this column return MORE THAN ONE answer because asking what angle rotation would result in the given trig values.

Calculus BC

Unit 1 Day 1

Polar Coordinates

An alternative way to plot points!

The Polar Express

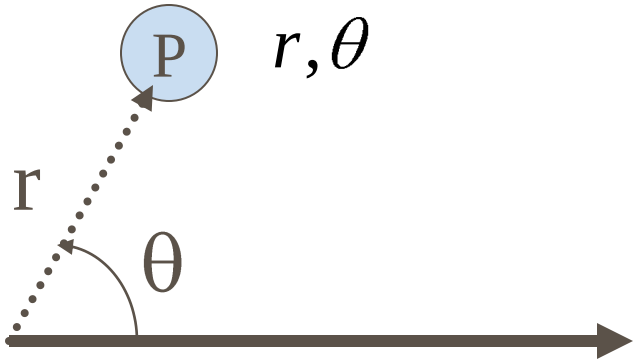
First we start with the **POLE** or **ORIGIN**

Then we extend a ray to the right. This is called the **POLAR AXIS**

Getting to the point…

The location of a point "P" is specified by:

1. Angle measure relative to the polar axis: θ
2. AND distance from the pole: r



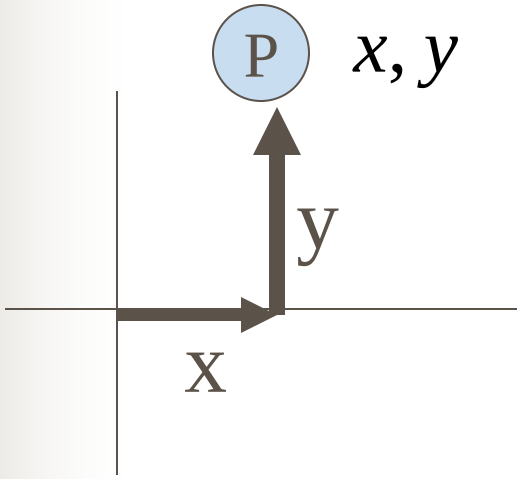
Compare

Cartesian

(x,y)

x = how far to go left or right

y = how far to go up or down.

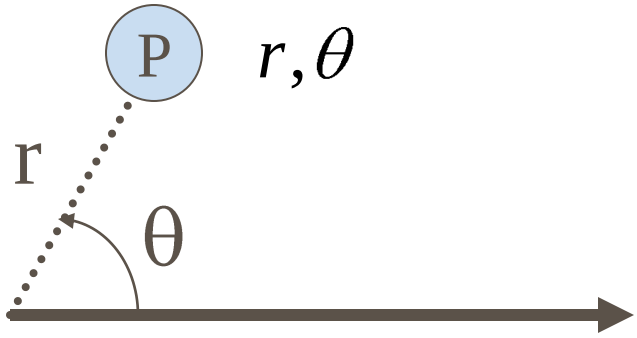


Polar

(r, θ)

r = distance from pole (radius)

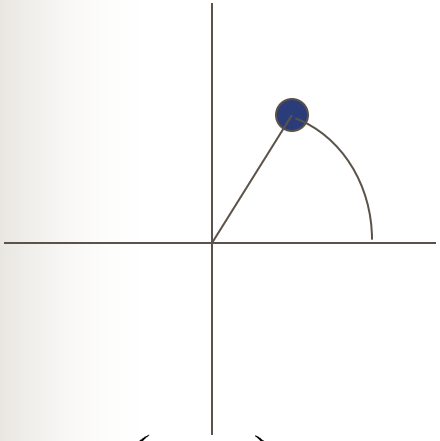
θ = the angle the radius makes with the polar axis



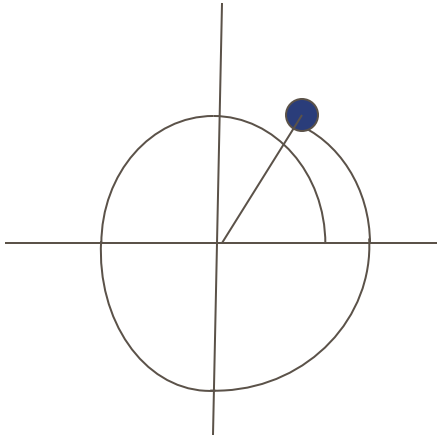
Cartesian vs Polar

- Cartesian Coordinate System
  - A point is named by a unique ordered pair (x,y)
- Polar Coordinate System
  - A point can be named by many different ordered pairs (r, θ)
  - Polar points are NOT unique!

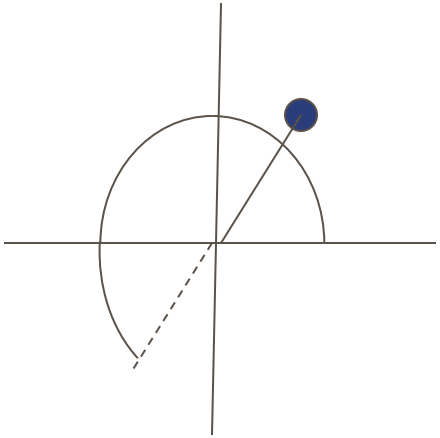
Let's explore . . .



$\left(5, \frac{\pi}{4}\right)$



$\left(5, \frac{9\pi}{4}\right)$



$\left(-5, \frac{5\pi}{4}\right)$

r, θ

$r, \theta + 2\pi n$

$-r, \theta \pm \pi$

$-r, \theta + \pi + 2\pi n$

Together let's plot the following polar points.

Plot AND label all 4 points on the same grid.

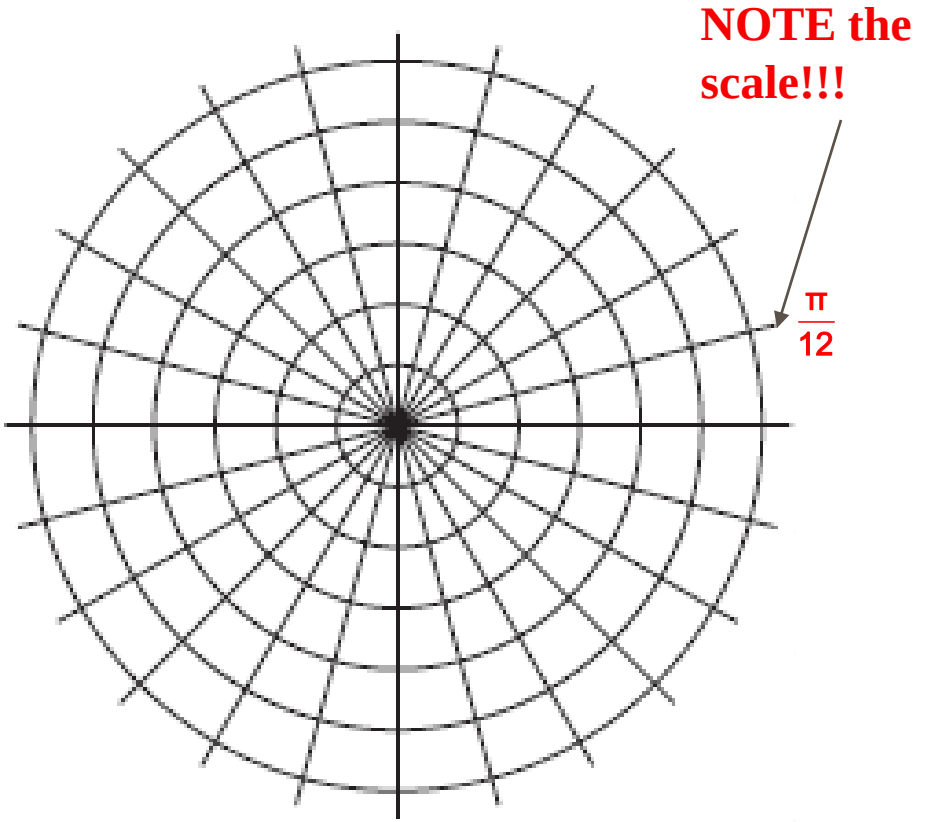
(Use the grids on Packet p.1)

$A\ \ 2, \pi$

$B\left(-3, \frac{\pi}{2}\right)$

$C\left(1, \frac{-\pi}{6}\right)$

$D\left(-3, \frac{5\pi}{6}\right)$



Together let's plot the following polar points.

Plot AND label all 4 points on the same grid.

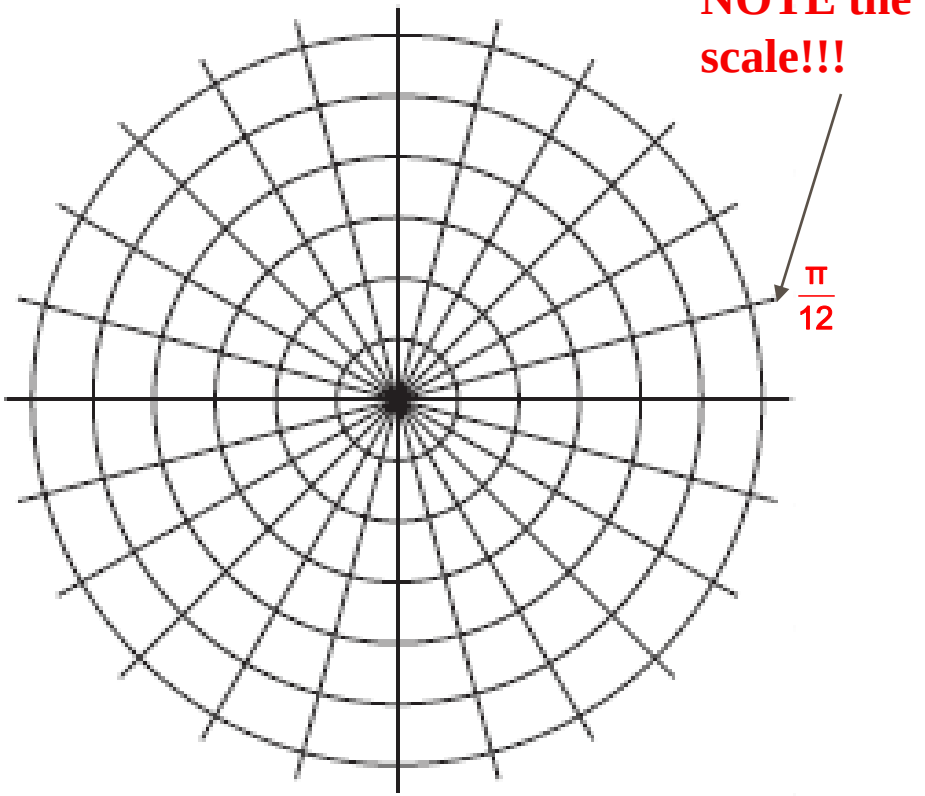
(Use the grids on Packet p.1)

$E\left(2, \frac{2\pi}{3}\right)$

$F\left(-1, \frac{\pi}{4}\right)$

$G\left(-2, \frac{5\pi}{4}\right)$

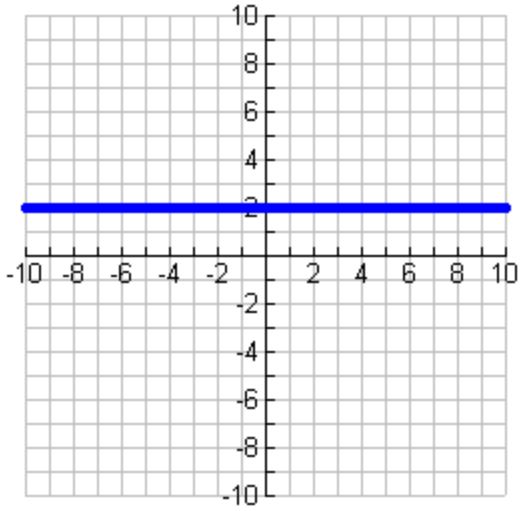
$H\ \ -3, 0$



Cartesian Coordinates

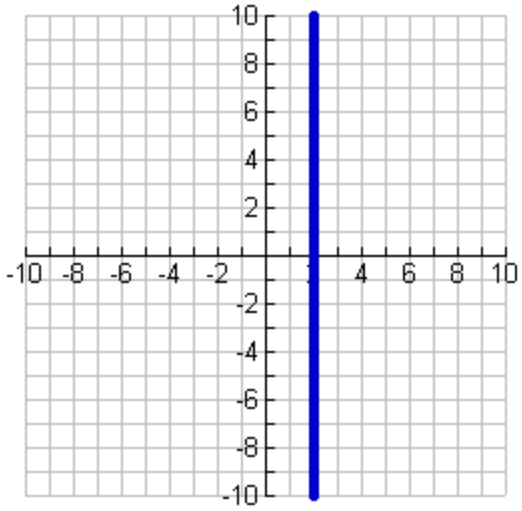
Single variable equations are special cases

$y = 2$



Horizontal line

$x = 2$

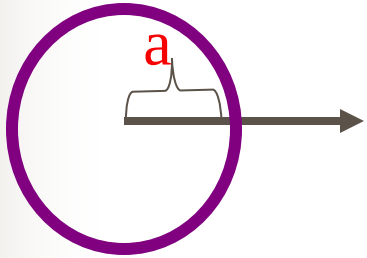


Vertical line

With Polar Coordinates

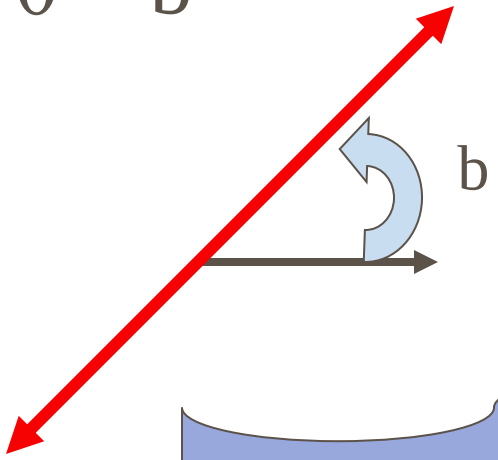
What happens when either r or θ is a constant?

r = a



Regardless of Angle of Rotation (θ) the Distance From the Pole (r) is constant

θ = b



Regardless of Distance from the Pole (r) the Angle of Rotation (θ) is constant

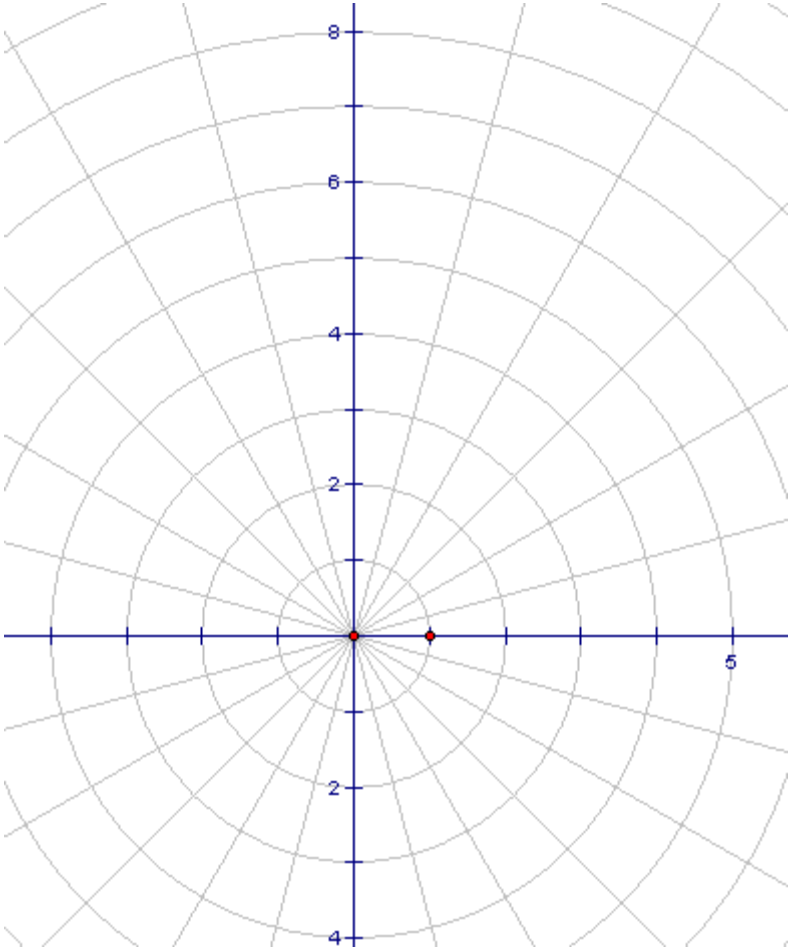
Plotting a Polar Equation using a Table of Values

$r = 8 \sin \theta$

θ	r
0	
$\pi/6$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$5\pi/6$	
π	

Hint:

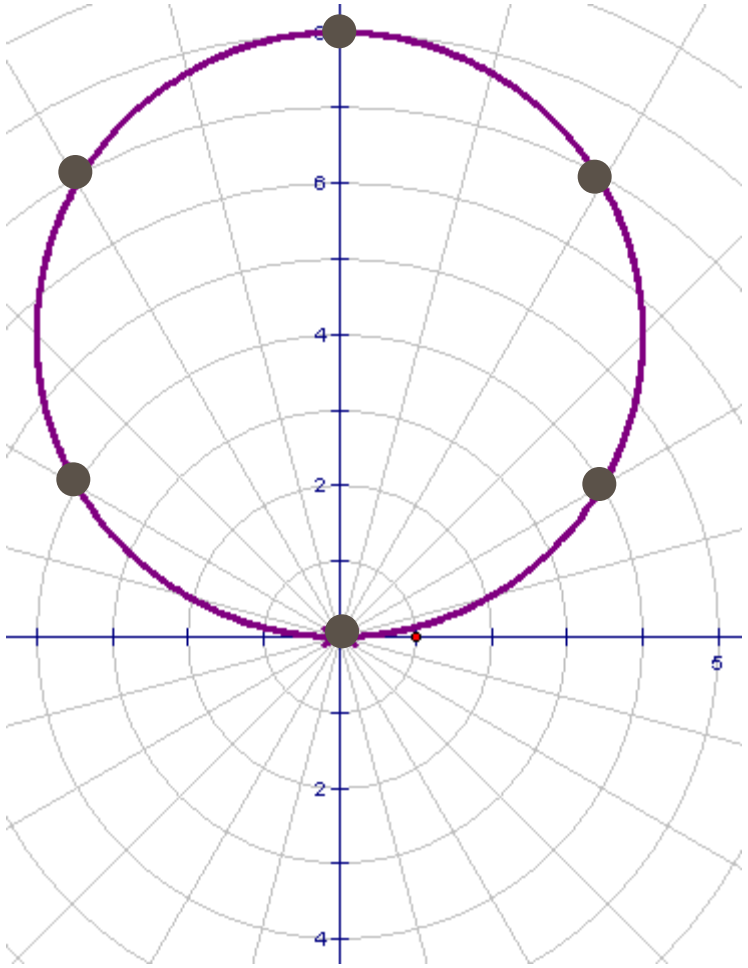
$4\sqrt{3} \approx 6.93$



Example of a Polar Equation

$r = 8 \sin \theta$

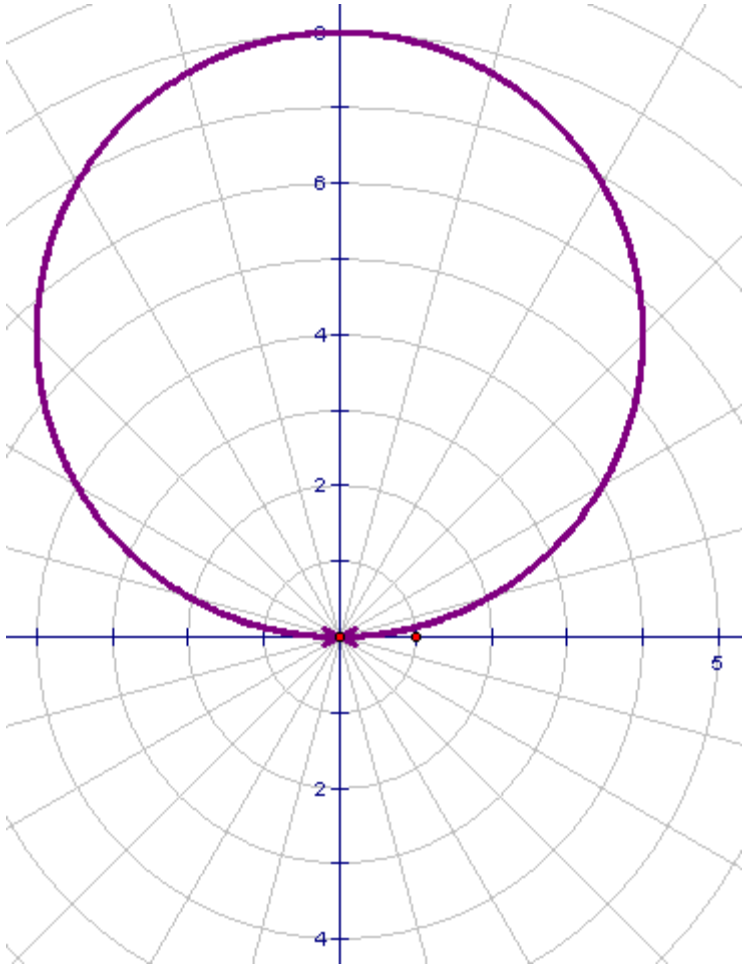
θ	r
0	0
$\pi /6$	4
$\pi /3$	6.93
$\pi /2$	8
$2\pi /3$	6.93
$5\pi /6$	4
π	0



Now continue on to 2π

$r = 8 \sin \theta$

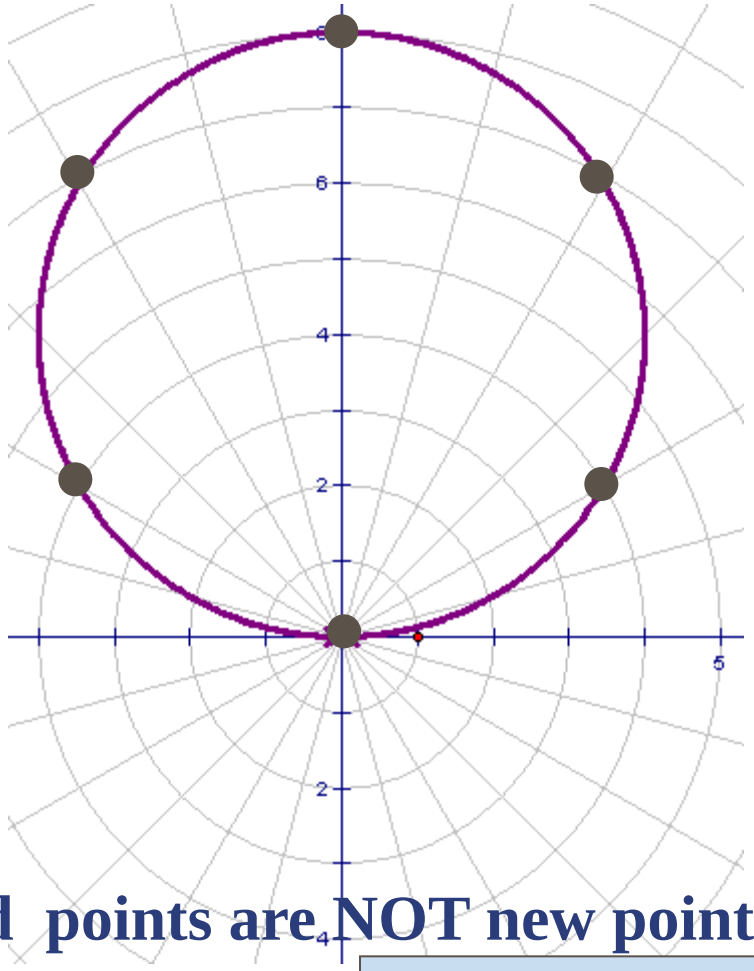
θ	r
π	0
7π /6	
4π /3	
3π /2	
5π /3	
11π /6	
2π	



Looking at our two sets of data points . . .

$r = 8 \sin \theta$

θ	r	θ	r
0	0	π	0
π /6	4	7π /6	-4
π /3	6.93	4π /3	-6.928
π /2	8	3π /2	-8
2π /3	6.93	5π /3	-6.928
5π /6	4	11π /6	-4
π	0	2π	0



$\therefore$ **The second set of plotted points are NOT new points!**

Why do you think this happens?

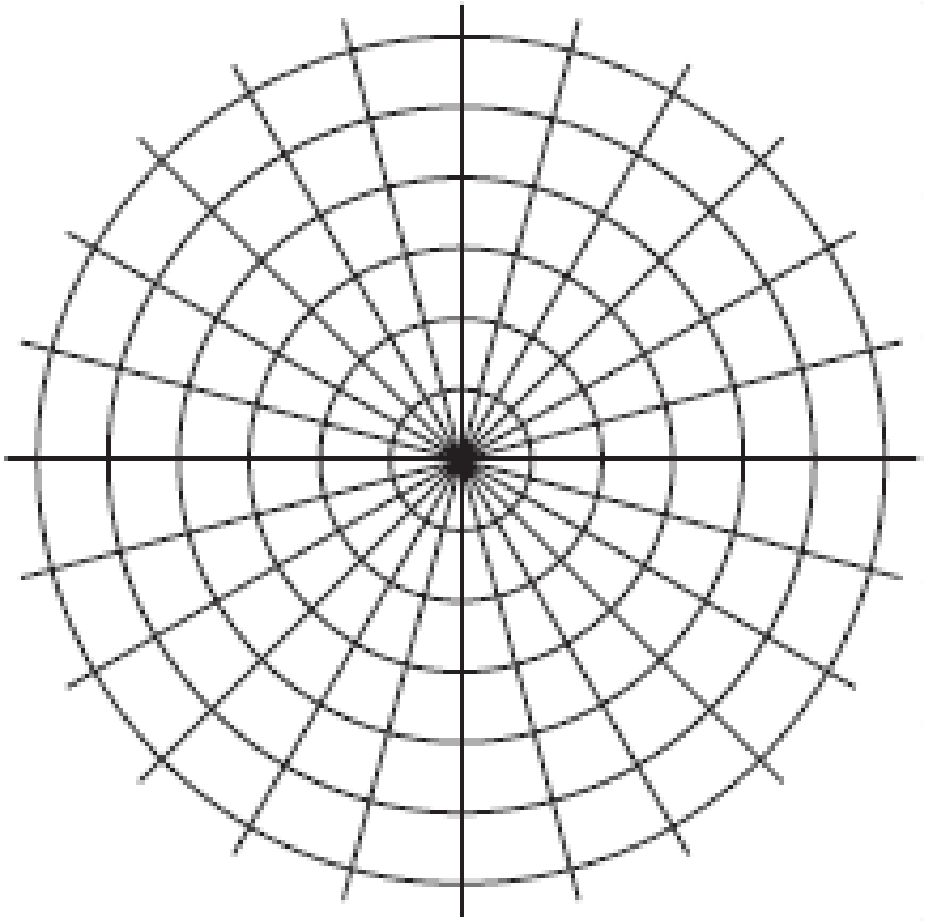
$$r, \theta = -r, \theta + \pi$$

Let's explore the next problems using the calculator . . .

- Polar Mode
- 2nd Format PolarGC
- For this problem set the window:
  - θmin=0, θmax=2π, θstep=π/12
  - xmin=-2, xmax=3, xscl=1
  - ymin=-2, ymax=2, yscl=1

Now for the equation . . .

$$r(\theta) = 1 + \cos\theta$$

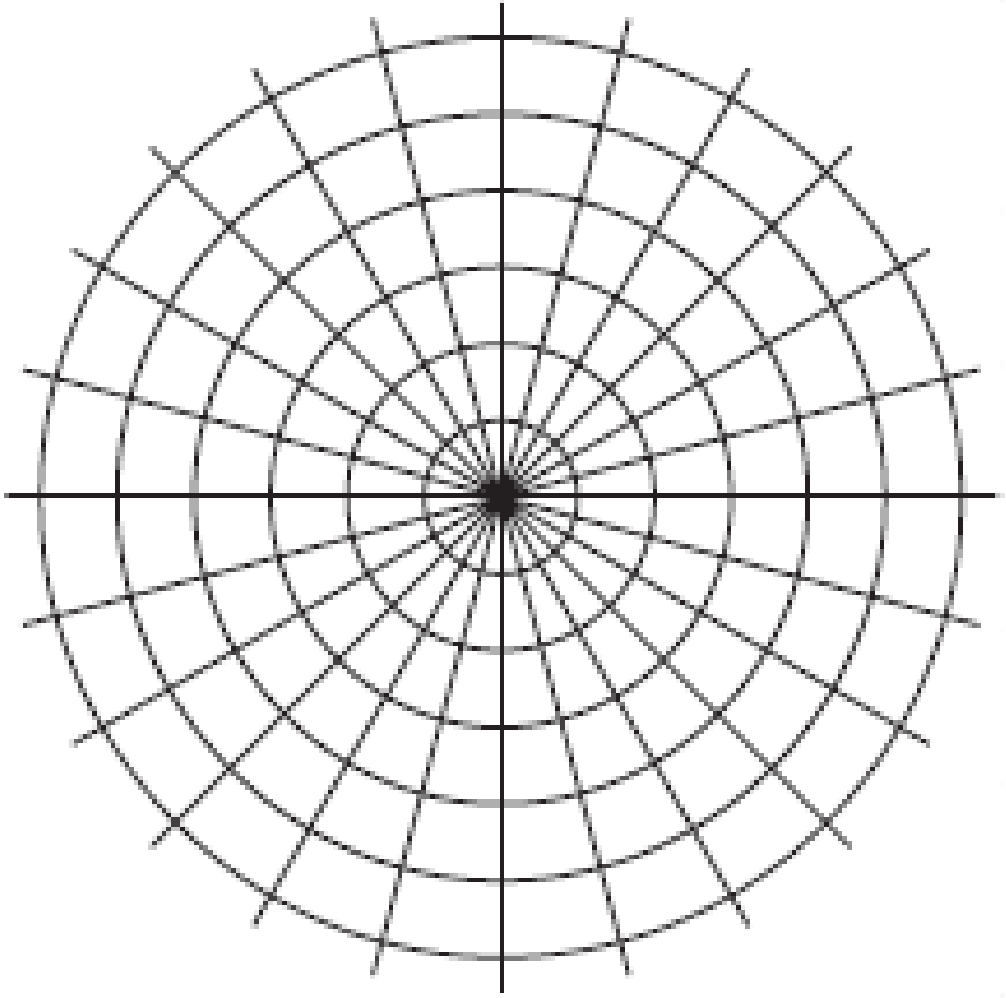


When do the points start repeating?

One more . . .

$r = \sin\ 3\theta$

To smooth out the curve make the θstep smaller.
θstep=0.1

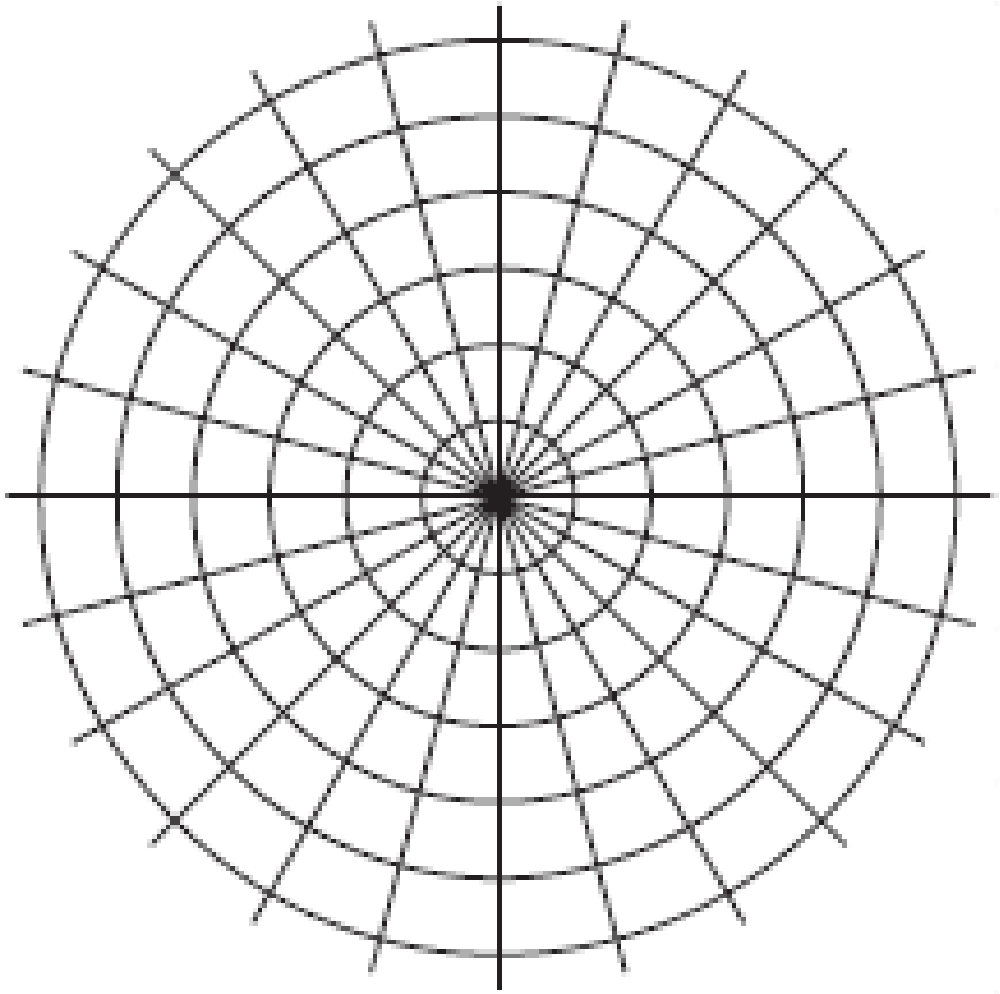


When do the points start repeating?

Last One . . .

$$r = 2 - \sin\theta$$

Adjust your window to get the graph to fit.



When do the points start repeating?