Limits Review

Review of Limit Techniques

- Direct substitution
- Factoring
- Multiply by 1
- A few to memorize

Direct Substitution:

If the function is *continuous* at *x* = *c* then use *direct substitution*.

$$\lim_{x \to c} f(x) = f(c)$$

Simply evaluate the function at *x* = *c*.

That is also the limit value.

Factoring

$$f(x) = \frac{x^2 + 3x - 4}{x + 4} \qquad g(x) = x - 1$$

Factor *f* and you get function *g*.
Functions *f* and *g* behave exactly the same everywhere except *x* = *-4*

f(-4) D.N.E., however *g*(-4) = -5

$$\lim_{x \to -4} f(x) = \lim_{x \to -4} g(x) = -5$$

Practice:

$f(x)=\frac{x^2-4}{x+2},\ \lim_{x\to -2} f(x)=$

$g(x)=\frac{x^3-x}{x},\ \lim_{x\to 0} g(x)=$

$h(x)=\frac{x^3-1}{x-1},\ \lim_{x\to 1} h(x)=$

Dealing with zero

$\frac{0}{nonzero} = 0$

$\frac{nonzero}{0} = \text{undefined}$

$\frac{0}{0} = \text{indeterminate}$

Undefined and indeterminate are NOT the same thing!

Undefined is an answer and means it does not exist.
Indeterminate means we do not know the answer…yet.

Use a common denominator

$ex)\ \lim_{t\to 0}\frac{6-\frac{1}{t}}{3+\frac{1}{t}}=$

You try: $\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

A couple Sine limits

1. $\lim_{x\to 0}\frac{\sin x}{x}=1$

2. $\lim_{x\to \infty}\frac{\sin x}{x}=0$

Ex)

1. $\lim_{x\to 0} \frac{\sin x}{5x} =$

2. $\lim_{x\to 0} \frac{\sin^2 x}{x} =$

Examples

3. $\lim_{x\to 0} \frac{x}{\sin x} =$

4. $\lim_{x\to 0} \frac{\sin 7x}{x} =$

A New Method – U substitution

$$\lim_{x\to\infty} \sin\left(\frac{1}{x}\right)$$

Let $u = \lim_{x\to\infty} \frac{1}{x}$

$u = 0^+$ approaching 0 from #s greater than 0

$\lim_{u\to 0^+} \sin(u)$ now substitute and evaluate

$= 0$

You try

$$\lim_{x\to\infty} 3\cos\left(\frac{1}{x}\right) + \frac{1}{x}$$

Let $u = \lim_{x\to\infty} \frac{1}{x}$

$u = 0^+$ approaching 0 from #s greater than 0

$\lim_{u\to 0^+} 3\cos(u) + u$ now substitute and evaluate

$= 3(1) + 0 = 3$

Get rid of negative exponents

$$\lim_{x\to\infty}\left(xe^{-x}\right)$$

$$\lim_{x\to\infty}\left(\frac{x}{e^{x}}\right) \quad = 0$$

Which is growing faster?

uSubstitution

uSubstitution:

$$\int (x^2+5)^3(2x)\ dx =$$

$$u = x^2 + 5$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\ dx$$

Another trig problem

$$\int \sin^3 x \cos x\ dx$$

We have 2 choices for *u*?

$u = \cos x$ X

$du = -\sin x\ dx$

$-du = \sin x\ dx$

$u = \sin x$

$du = \cos x\ dx$

The higher powers will usually be *u*.

$$= \int u^3\ du \qquad = \frac{u^4}{4} + c \qquad = \frac{\sin^4 x}{4} + c$$

A little bit harder now

$$\int \frac{\sin x}{\cos x}\,dx$$

What do we make u?

~~$u = \sin x$~~

~~$du = \cos x\,dx$~~ ✗

$\cos x$ is not on the bottom cannot use this.

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$-\int \frac{1}{u}\,du$$

$$= -\ln|u| + c = -\ln|\cos x| + c$$

Integration by Parts

$\int x^3 \sin x\,dx$

$\int \ln x\,dx$

Integration and Partial Fractions

$$\int \frac{1}{x^2-5x+6}\,dx$$

$$\frac{1}{(x-3)(x-2)}=\frac{A}{x-3}+\frac{B}{x-2}$$

$$1=A(x-2)+B(x-3)$$

Let $x=2$	$x=3$
$1=A(2-2)+B(2-3)$	$1=A(3-2)+B(3-3)$
$1=0-B$	$1=A+0$
$B=-1$	$A=1$

$$\int \frac{1}{x-3}-\frac{1}{x-2}\,dx=\ln|x-3|-\ln|x-2|+c$$

Solve $\int \frac{x+2}{x^2-4x}\,dx$

$$\frac{x+2}{x(x-4)}=\frac{A}{x}+\frac{B}{x-4}$$

$$x+2=A(x-4)+Bx$$

Let $x=0$	$x=4$
$0+2=A(0-4)+B(0)$	$4+2=A(4-4)+B(4)$
$2=-4A$	$6=4B$
$A=-\frac{1}{2}$	$B=\frac{3}{2}$

$$\int \frac{-1/2}{x}+\frac{3/2}{x-4}\,dx=-\frac{1}{2}\ln|x|+\frac{3}{2}\ln|x-4|+c$$

Solve $\int \frac{6x+7}{x^2+4x+4}\,dx = \int \frac{6x+7}{(x+2)(x+2)}\,dx$

Repeated Factors