Warmup—Calculator ALLOWED
Remember to put these in your NOTES!

1. If $y = 5x^2 + 4x$ and $x = \ln t$ then $\frac{dy}{dt} = ?$

2. The area enclosed by $r(\theta) = 4 + \cos\theta$
for $0 \le \theta \le 2\pi$

How many days until the AP Calculus Exam

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1. If $y = 5x^2 + 4x$ and $x = \ln t$ then $\frac{dy}{dt} = ?$

ANSWER: $\dfrac{10\ln t + 4}{t}$

2. The area enclosed by $r(\theta) = 4 + \cos\theta$
for $0 \le \theta \le 2\pi$

ANSWER: 51.836

What if you did not have a calculator?

How would you integrate

$$\frac{1}{2}\int_0^{2\pi} (4 + \cos\theta)^2 \, d\theta$$

Today's Topic:
Power Series

Geometric Series Review:

Each term in a geometric series is found by multiplying the preceding term by the same **number**, *r*.

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + \cdots = \sum_{n=1}^{\infty} ar^{n-1}$$

This converges to $\frac{a}{1-r}$ if $|r| < 1$, and diverges if $|r| \ge 1$.

$|r| < 1$ And $-1 < r < 1$
are mathematically equivalent statements

Thus, a geometric series converges if $-1 < r < 1$.

New Terminology :

$-1 < r < 1$ is the **interval of convergence**.

Interval of convergence:
Set of numbers for which the series converges

More NEW Vocabulary

RADIUS OF CONVERGENCE (R):

Distance from the center of interval of convergence to the endpoint

Moving on from :

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + ... + ar^{n-1} + ...$$

If r were a **variable** instead of a constant the series would look like . . . .

$$f(x) = \sum_{n=1}^{\infty} ax^{n-1} = a + ax + ax^2 + ax^3 + ... + ax^{n-1} + ...$$

Furthermore . . . .

The coefficients of each term can be different and the starting n value can be 0…

$$f(x) = \sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + ... + C_{n-1} x^{n-1} + C_n x^n ...$$

This is called a **POWER SERIES**

Power Series Notes . . .

A power series is a type of geometric series, where each term is obtained by multiplying by a variable or variable phrase.

A power series "CENTERED at 0" would look like:

$$f(x) = \sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + ... + C_{n-1} x^{n-1} + C_n x^n ...$$

A power series "CENTERED at x=a" would look like:

$$f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1 (x-a) + ... + C_n (x-a)^n ...$$

Which of the following are power series?

If yes, identify:

1. What defines the coefficients
2. What is the variable or variable phrase

$1. \sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$ $\quad 2. \sum_{n=2}^{\infty} \frac{x^n}{2 \ln n}$

$3. \sum_{n=0}^{\infty} \frac{3^n}{2^{2n}}$ $\quad 4. \ 1 - x^2 + x^4 - x^6 + ...$

Use the Ratio Test to Determine Power Series Convergence

RATIO TEST: Given $\sum_{n=0}^{\infty} b_n$

if $\lim_{n \to \infty} \left| \frac{b_{n+1}}{b_n} \right| < 1$ then $\sum_{n=0}^{\infty} b_n$ Converges

if $\lim_{n \to \infty} \left| \frac{b_{n+1}}{b_n} \right| > 1$ then $\sum_{n=0}^{\infty} b_n$ Diverges

if $\lim_{n \to \infty} \left| \frac{b_{n+1}}{b_n} \right| = 1$ Inconclusive

There are 3 possible outcomes:

1) Converge for some interval $|x-a|<R$,
   where *a* is center of the interval and
   R is the "radius of convergence"

2) Converge only when x = a (a is some number)

3) Converge for all real #s

Examples of the three cases

Case 1) Converge for some interval of *x* values.
NOTE: Requires checking endpoints of the interval.

$$f(x)=\sum_{n=1}^{\infty}\frac{(x-3)^n}{n}$$

Interval of Convergence: ______________

Radius of Convergence: ______________

Case 2) Converge only for single *x* value

$$f(x)=\sum_{n=0}^{\infty}n!x^n$$

Interval of Convergence: ______________

Radius of Convergence: ______________

Case 3) Converge for any *x* value

$$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

This is a type of Bessel Function

Among other applications, Bessel functions are used to model vibrating surfaces like drums and heat conduction of surfaces like plates.

Interval of Convergence: ______________

Radius of Convergence: ______________

Classwork – p.1(evens)