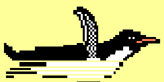
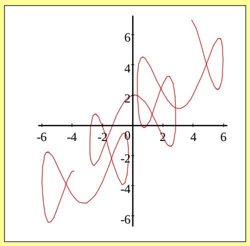
Parametric Functions





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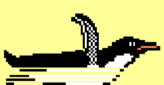
HW Policy

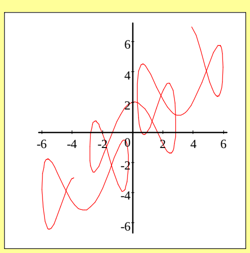
- If you earned an A or B
  - TURNING IN Homework is optional
- If you did not earn an A or B
  - Homework will be collected DAILY
  - Must be neat, complete, organized and on separate paper
- COMPLETING Homework is NOT Optional!!!

Parametric Functions



Background Info—FIRST

No need for notes yet!!!



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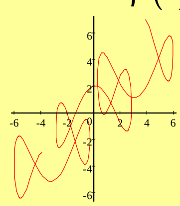
Parametric equations are used to describe motion

$$x = f(t) \qquad y = g(t)$$



This curve is:

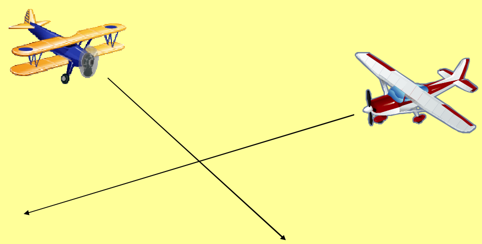
$$x(t) = t + \sin 2t$$
$$y(t) = t + 2\cos(5t)$$

Position of an object @ time t is $(x(t), y(t))$

→

Curves created by a pair of parametric equations can be treated as the graph of a function of the parameter *t* because each value of *t* produces a unique point on the curve.

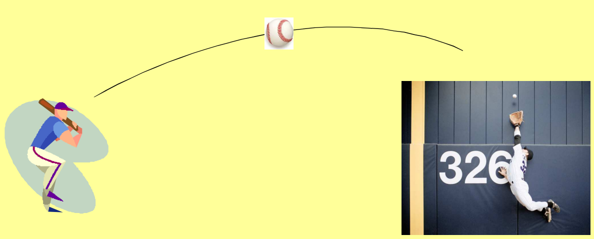
Parametric equations could be used to determine if the flight paths of two planes would result in a midair crash.



If $(x(t), y(t))$, the horizontal and vertical positions, of each plane is the same at the same *t-value* there will be a CRASH!

Parametrics can tell us the horizontal distance a ball has traveled $x = (150\cos 18)t$

AND what the height of the ball is at a specified time $y = (150\sin 18)t + 3 - 16t^2$



Parametrics Equations

- Parametrics tell us more about position than a typical xy-equation
- x and y are dependent on a third variable, often known as *t* (time), instead of being defined in terms of each other

$$x^2 + y^2 = 4$$ This equation can answer "where"

$$x = 2\cos t$$
$$y = 2\sin t$$
Parametric equations can answer "where" and "when"

BOTH equations are of a circle w/ center (0,0) and a radius of 2.

Finding the slope of a parametrized curve:

We want to find $\frac{dy}{dx}$

But remember the functions are in terms of t, not x.

Here is how . . .

$\frac{dy}{dt} \div \frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dx}$ This is what we want.

Conclusion: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Look familiar?

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Parametric

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

Polar

Example:
Finding the slope of a parametrized curve defined by the equations $x = \sin t$

$y = t^2 + 1$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Solution

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$x = \sin t$

$y = t^2 + 1$

Find $\frac{dy}{dx}$

$\frac{dx}{dt} = \cos t$ $\frac{dy}{dt} = 2t$

$$\frac{dy}{dx} = \frac{2t}{\cos t}$$

You Try:

Find dy/dx. Then write the equation of the line tangent to the curve at $t = \frac{\pi}{4}$

x = 4 sin t

y = 2 cos t

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

A new type of example:

Find the value of the slope of the curve at the *point* (-2,8)

$$x = t^2 - 3t \quad y = t^3$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Find the ***points*** at which the tangent to the curve is a)Vertical b) Horizontal

HW problem #7:

x =2+cos(t)

y = -1+sin(t)

BRAINSTORM—What do you think we should do?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

The second derivative of a parametrized curve, is the derivative of the first derivative.

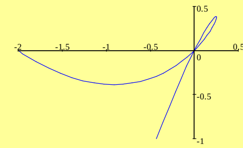
But since the first derivative is in terms of t

and we are looking for $\frac{d^2y}{dx^2}$:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{d\,\frac{dy}{dx}}{\frac{dx}{dt}}$$

Find $\frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{\frac{d\,\frac{dy}{dx}}{dt}}{\frac{dx}{dt}}$.

Given $x = t - t^2$ and $y = t - t^3$.

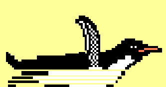


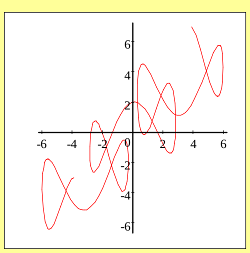
1. Express *dy/dx* in terms of t

2. Find the derivative of *dy/dx* with respect to t.

3. Divide by *dx/dt*.

Questions so far?





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The length of a parametrized curve:

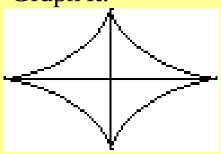
$$So, \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

(Notice the similarity to the distance formula.)

Example: Find the length of the following curve

$$x = \cos^3 t, \qquad y = \sin^3 t, \qquad 0 \le t \le 2\pi$$

Graph it:



$$x = \cos^3 t, \qquad y = \sin^3 t, \qquad 0 \le t \le 2\pi$$

$$\left(\frac{dx}{dt}\right)^2 =$$

$$\left(\frac{dy}{dt}\right)^2 =$$

$$So, \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$