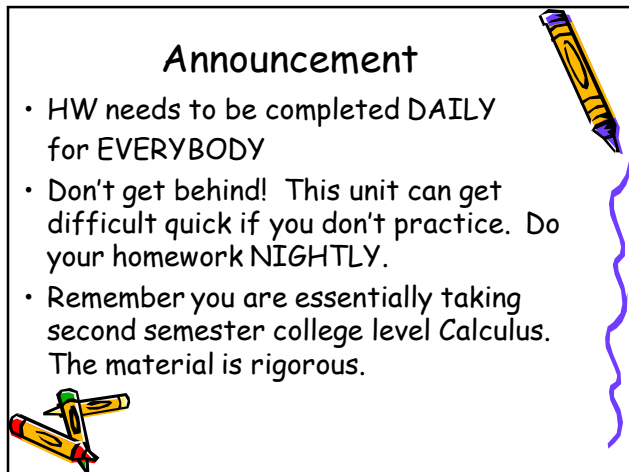


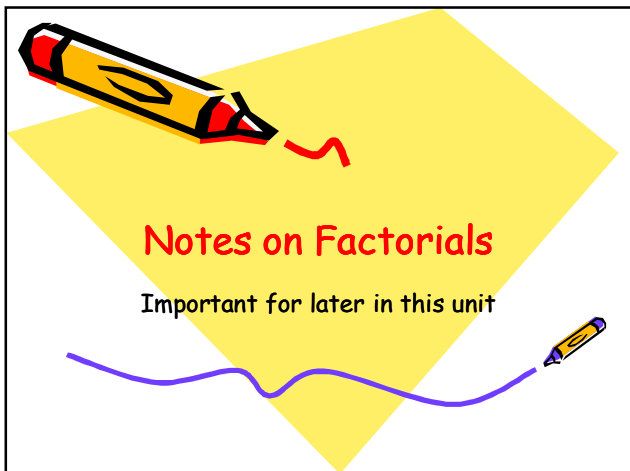
**Calculus BC Unit 5**

Day 1 Agenda:  
Notes on Factorials



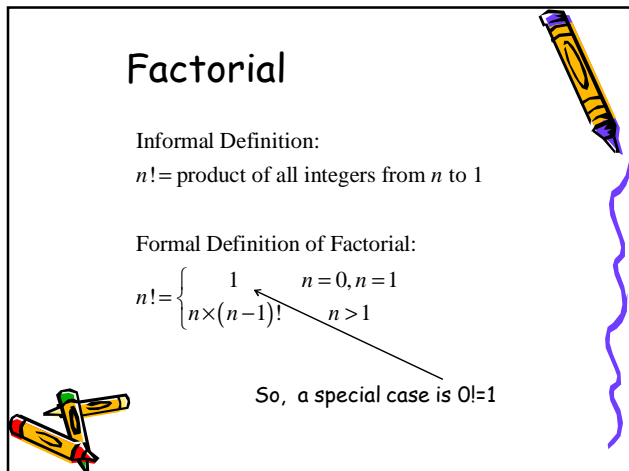
### Announcement

- HW needs to be completed DAILY for EVERYBODY
- Don't get behind! This unit can get difficult quick if you don't practice. Do your homework NIGHTLY.
- Remember you are essentially taking second semester college level Calculus. The material is rigorous.



**Notes on Factorials**

Important for later in this unit



### Factorial

Informal Definition:  
 $n!$  = product of all integers from  $n$  to 1

Formal Definition of Factorial:

$$n! = \begin{cases} 1 & n = 0, n = 1 \\ n \times (n-1)! & n > 1 \end{cases}$$

So, a special case is  $0! = 1$

## Examples

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

$$10! = 10 \cdot 9 \cdot 8 \cdots 3 \cdot 2 \cdot 1 = 3,628,800$$

$$20! = 20 \cdot 19 \cdot 18 \cdots 2 \cdot 1 = 2.432 \times 10^{18}$$



## Factorial on the Calculator

Enter the number first, then type

MATH  $\rightarrow$  PRB  $\rightarrow$  Option 4

$$30! = 2.652 \times 10^{32}$$

$$40! = 8.159 \times 10^{47}$$

$$50! = 3.041 \times 10^{64}$$



## Up for a race?

Who can determine the largest integer the calculator can find the factorial for?

We know it can do 50!

$$69! = 1.711 \times 10^{98}$$

$$70! = \text{Error}$$



## Brainstorm: Evaluate the limits

$$\lim_{n \rightarrow \infty} \frac{e^n}{n!} =$$

$$\lim_{n \rightarrow \infty} \frac{n!}{e^n} =$$



## Which is growing faster?

$$\lim_{n \rightarrow \infty} e^n \text{ or } \lim_{n \rightarrow \infty} n!$$

$$e^n = e \cdot e \cdots e \cdot e \cdots e \cdot e \cdots$$

$$n! = 1 \cdot 2 \cdot 3 \cdots 10 \cdots 100 \cdots 1000 \cdots$$

The factors of  $e^n$  are all  $e$ .

The factors of  $n!$  are increasing without bound.

$$\lim_{n \rightarrow \infty} n! > \lim_{n \rightarrow \infty} e^n$$



## Conclusion

$$\lim_{n \rightarrow \infty} \frac{e^n}{n!} = 0$$

$$\lim_{n \rightarrow \infty} \frac{n!}{e^n} = \infty$$



## Expanding Factorials

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

$$(n-1)! = (n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

$$(n-2)! = (n-2) \cdots 3 \cdot 2 \cdot 1$$

$$(2n+1)! = (2n+1)(2n)(2n-1) \cdots 3 \cdot 2 \cdot 1$$



## More examples

$$7! = (7)6!$$

$$7! = (7 \cdot 6)5!$$

$$7! = (7 \cdot 6 \cdot 5)4!$$

$$n! = (n)(n-1)!$$

$$n! = (n)(n-1)(n-2)!$$

$$n! = n(n-1)(n-2)(n-3)!$$



## Simplifying Factorials

$$\frac{5!}{7!} = \frac{5!}{(7 \cdot 6)5!} = \frac{1}{7 \cdot 6}$$

$$\frac{8!}{11!} = \frac{8!}{(11 \cdot 10 \cdot 9)8!} = \frac{1}{11 \cdot 10 \cdot 9}$$

$$\frac{59!}{57!} = \frac{(59 \cdot 58)57!}{57!} = \frac{58 \cdot 59}{1}$$

Expand the larger factorial until you have the same factorial in the numerator and denominator.

Then the factorials will divide out.



## Simplifying factorials

$$\frac{(n-2)!}{n!} =$$

Before you start think about which factorial is bigger.

$$\frac{(n-2)!}{n(n-1)(n-2)!} = \frac{1}{(n-1)n}$$



## A trickier example

$$\frac{(2n-1)!}{(2n+1)!} =$$

$$\frac{(2n-1)!}{(2n+1)(2n)(2n-1)!}$$

$$= \frac{1}{(2n)(2n+1)}$$

Before you start think about which factorial is bigger.



## Why all the fuss about factorials?

$$f(x) = x^n$$

$$f'(x) = nx^{n-1}$$

$$f''(x) = n(n-1)x^{n-2}$$

$$f'''(x) = n(n-1)(n-2)x^{n-3}$$

$$f^n(x) = n(n-1)(n-2) \cdots 2 \cdot 1 = n!$$



## Try It!

$f(x) = x^4$  Find  $f^4(x)$  (the 4th derivative)

$$f' = 4x^3$$

$$f'' = 4 \cdot 3x^2$$

$$f''' = 4 \cdot 3 \cdot 2x^1$$

$$f^4 = 4 \cdot 3 \cdot 2 \cdot 1 = 4!$$



## Again

$$f(x) = x^{100}$$

$$f^{100}(x) = 100!$$



## And lastly

$$f(x) = 4x^{50}$$

$$f^{50}(x) = 4(50!)$$



Review/Correct of Unit 4 Test



UNUSED



$$n! = \begin{cases} 1 & n=0, n=1 \\ n \times (n-1)! & n > 1 \end{cases}$$

Using the formal definition

$$\begin{aligned} 4! &= (3!) \cdot 4 \\ &= (2! \cdot 3) \cdot 4 \\ &= (1! \cdot 2) \cdot 3 \cdot 4 \\ &= 1 \cdot 2 \cdot 3 \cdot 4 \end{aligned}$$

Conclusion:  $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

