

Unit 5 Review Day

Also posted on Blackboard:
Limit summary

Given $\sum_{n=1}^{\infty} a_n$

n^{th} term test

$$\lim_{n \rightarrow \infty} a_n = \begin{cases} \neq 0 & DV \\ 0 & \text{Inconclusive} \end{cases}$$

Ratio test

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \begin{cases} < 1 & CV \\ > 1 & DV \\ 1 & \text{Inconclusive} \end{cases}$$

Limit Comparison test

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \begin{cases} 0 & CV \text{ if } b_n \text{ CV} \\ \text{non-zero constant} & \text{Same as } b_n \\ \infty & DV \text{ if } b_n \text{ DV} \end{cases}$$

Root test

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \begin{cases} < 1 & CV \\ > 1 & DV \\ 1 & \text{Inconclusive} \end{cases}$$

Integral test

$$\lim_{n \rightarrow \infty} \int_1^n a_n = \begin{cases} \text{Any constant} & CV \\ \infty & DV \end{cases}$$

Activity:

Each day, thousands of people put their lives in the hands of doctors.

These miracle workers thrive in a high stress, high stakes environment.

One wrong decision and lives are destroyed.

Do you have what it takes to compete in ...



Match wits with the doctors of Seattle Grace Hospital.

Examine the patients' test results and make the correct diagnosis.

Patients

a_n = Your unknown patient

c_n = Known patient that converges

d_n = Known patient that diverges

Possible Diagnoses

Converge

Diverge

Inconclusive

Contradiction



Tests

$$\lim_{n \rightarrow \infty} a_n = \frac{1}{2}$$

Interpretation

Test for Divergence

(a.k.a. n^{th} term test.)

Limit not 0, therefore DV



Diagnosis

Diverge



Tests

$$\lim_{n \rightarrow \infty} a_n = 0$$

Interpretation

n^{th} term test. Limit = 0, inconclusive

$$\lim_{n \rightarrow \infty} \int a_n = 2$$

Integral Test. Integral CV, therefore so does series.



Diagnosis

Converge



Tests

$$\lim_{n \rightarrow \infty} a_n = 0$$

Interpretation

n^{th} Term Test. Limit is 0, inconclusive

$$\lim_{n \rightarrow \infty} \int a_n = \infty$$



Integral Test. Integral diverges, DV







Diagnosis

Diverge



<u>Tests</u> $\lim_{n \rightarrow \infty} a_n = 0$ $\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = 3$	<u>Interpretation</u> n^{th} Term Test. Limit is 0, inconclusive Limit Comparison Test. Limit is non-zero constant, same behavior as C.
	<u>Diagnosis</u> Converge 

<u>Tests</u> $\lim_{n \rightarrow \infty} a_n = 0$ $\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = 0$	<u>Interpretation</u> n^{th} Term Test. Limit is 0, inconclusive Limit Comparison Test. Limit is zero. Known series diverges. Inconclusive.
	<u>Diagnosis</u> Inconclusive 

<u>Tests</u> $\lim_{n \rightarrow \infty} a_n = \frac{1}{3}$ $\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = 0$	<u>Interpretation</u> n^{th} Term Test. Limit is non-zero, DV Limit Comparison Test. Limit is zero. Known series converges. CV.
	<u>Diagnosis</u> Contradiction 

Can't resolve contradictions.
At least one of the tests must have been done incorrectly.
With the information given, there is no way to know which one.
Remember to use multiple tests, to help prevent mistakes.
If they do just one test and make a mistake, your patient could be burned toast.
At least with a contradiction, you know something was done incorrectly.

Multiple Tests and Contradictions

If you do one test, but do it incorrectly, the problem is wrong.



If you do multiple tests and get the same conclusion, you have high confidence in the answer.

If you do multiple tests and get conflicting conclusions, at least one of the tests was done incorrectly. You'll have to figure out which one, but at least you know there is a mistake.



Moral

If you have the time, confirm your conclusions by doing a different type of test.



<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = \infty$	Limit Comparison Test. Limit is infinite. Known series diverges. DV.

	<u>Diagnosis</u> Diverge	
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<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} \int a_n = 3$	Integral Term Test. Integral converges. CV.
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = 3$	Limit Comparison Test. Limit is non-zero constant. Same behavior, known series diverges. DV.

	<u>Diagnosis</u> Contradiction	
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<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 2$	Ratio Test. Limit is greater than 1. DV

	<u>Diagnosis</u> Diverge	
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<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 1$	n^{th} Term Test. Limit is non-zero, DV.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$	Ratio Test. Limit is 1. Inconclusive.


	<u>Diagnosis</u> Diverge	
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<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \frac{1}{2}$	Ratio Test. Limit is less than 1. CV.




Diagnosis

Converge




<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 1$	n^{th} Term Test. Limit is non-zero, DV.
$\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = 1$	Limit Comparison Test. Limit is non-zero constant, same behavior. Known series converges. CV.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$	Ratio Test. Limit is 1. Inconclusive.




Diagnosis

Contradiction




<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = 1$	Limit Comparison Test. Limit is non-zero constant, same behavior. Known series diverges. DV.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 2$	Ratio Test. Limit is greater than 1. DV.




Diagnosis

Diverge

DIVERGE






<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = 0$	Limit Comparison Test. Limit is zero. Known series converges. CV.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 0$	Ratio Test. Limit is less than 1. CV.







Diagnosis



Converge




<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = \infty$	Limit Comparison Test. Limit is infinite. Known series converges. Inconclusive.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \frac{1}{2}$	Ratio Test. Limit is less than 1. CV.
	<u>Diagnosis</u> Converge 

<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = \infty$	Limit Comparison Test. Limit is infinite. Known series diverges. DV.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 3$	Ratio Test. Limit is greater than 1. DV.
	<u>Diagnosis</u> Diverge 

<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = \frac{1}{2}$	Limit Comparison Test. Limit is non-zero constant. Same behavior. Known series diverges. DV.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \frac{1}{2}$	Ratio Test. Limit is less than 1. CV.
	<u>Diagnosis</u> Contradiction 


<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = \frac{1}{3}$	n^{th} Term Test. Limit is non-zero, DV.
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = 0$	Limit Comparison Test. Limit is zero. Known series diverges. Inconclusive.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$	Ratio Test. Limit is 1. Inconclusive.
	<u>Diagnosis</u> Diverge 

<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = \infty$	n^{th} Term Test. Limit is non-zero, DV.
$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = \infty$	Limit Comparison Test. Limit is infinite. Known series diverges. Diverge.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \infty$	Ratio Test. Limit is greater than 1. DV.




Diagnosis

Diverge



<u>Tests</u>	<u>Interpretation</u>
$\lim_{n \rightarrow \infty} a_n = 0$	n^{th} Term Test. Limit is zero, inconclusive.
$\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = 5$	Limit Comparison Test. Limit is non-zero constant. Same behavior. Known series converges. Converge.
$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$	Ratio Test. Limit is 1. Inconclusive.



Diagnosis

Converge

